ECON 626: Applied Microeconomics

Lecture 9:

Permutation tests (randomization inference) and Wild Cluster Bootstrap

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- This extends naturally to the case where treatment assignments are restricted in some way. Recall, for example, the Bruhn and McKenzie (2009) discussion of the many different restrictions that can be used to yield balanced randomization.

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- Divide the counter by the number of iterations. You have a p-value!

Gerber and Green, p.63: "... the calculation of p-values based on an inventory of possible randomizations is called *randomization inference*."

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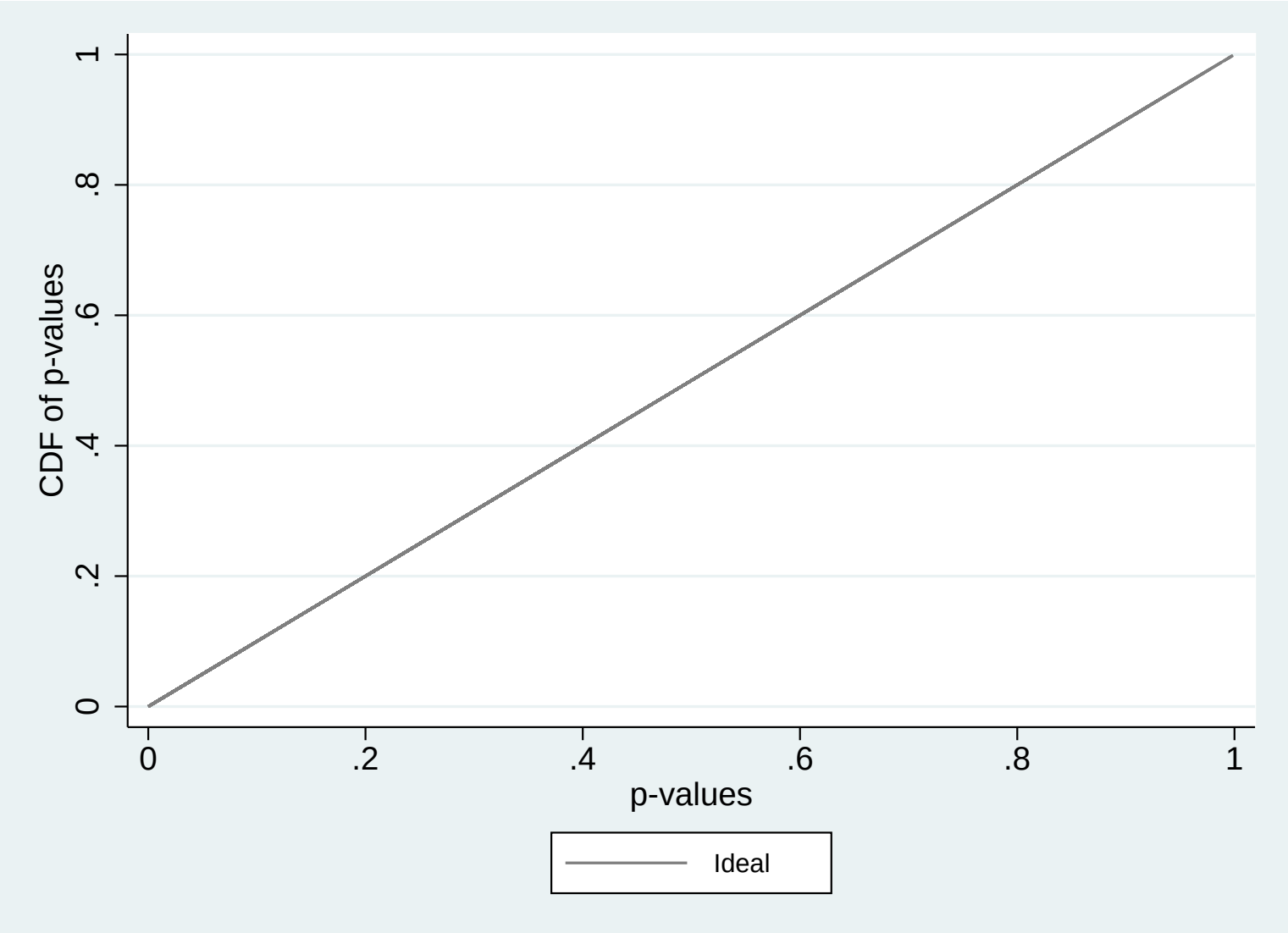
Or what if it were ten cups, and she got 4 out of 5?
This becomes unwieldy to calculate exactly. Activity: randomly sample.

Few Clusters:
Wild Cluster Bootstrap

What is the problem with having too few clusters?



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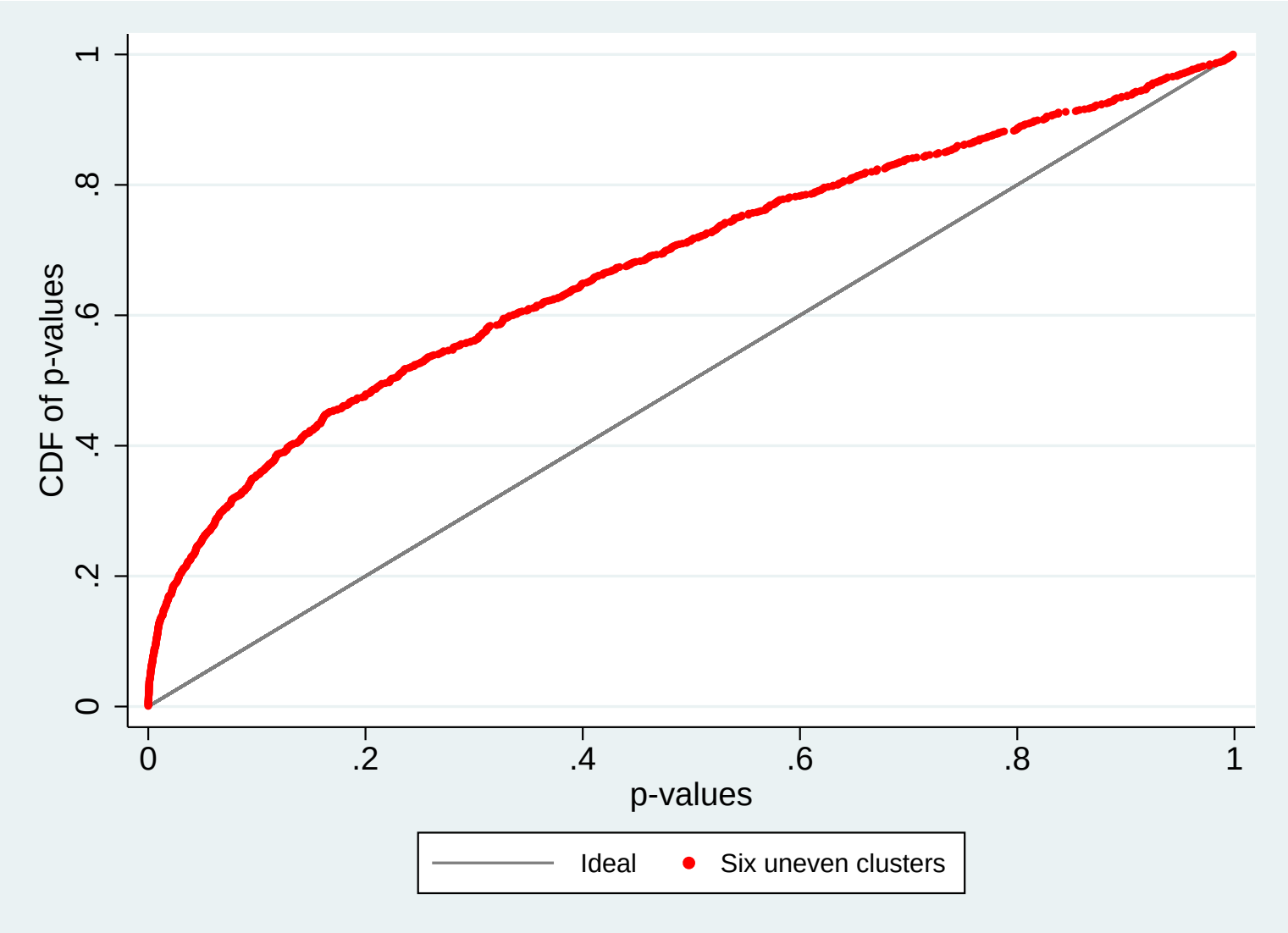

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Cameron, Gelbach, and Miller procedure goes as follows (Cameron and Miller 2015, Section VI.C.2):

- "First, estimate the main model, imposing (forcing) the null hypothesis that you wish to test... For example, for test of statistical significance of a single variable regress y_{ig} on all components of x_{ig} except the variable that has regressor with coefficient zero under the null hypothesis."

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- "Form the residual $\tilde{u}_{ig} = y_i g - x'_{ig}\tilde{\beta}_{H0}$"

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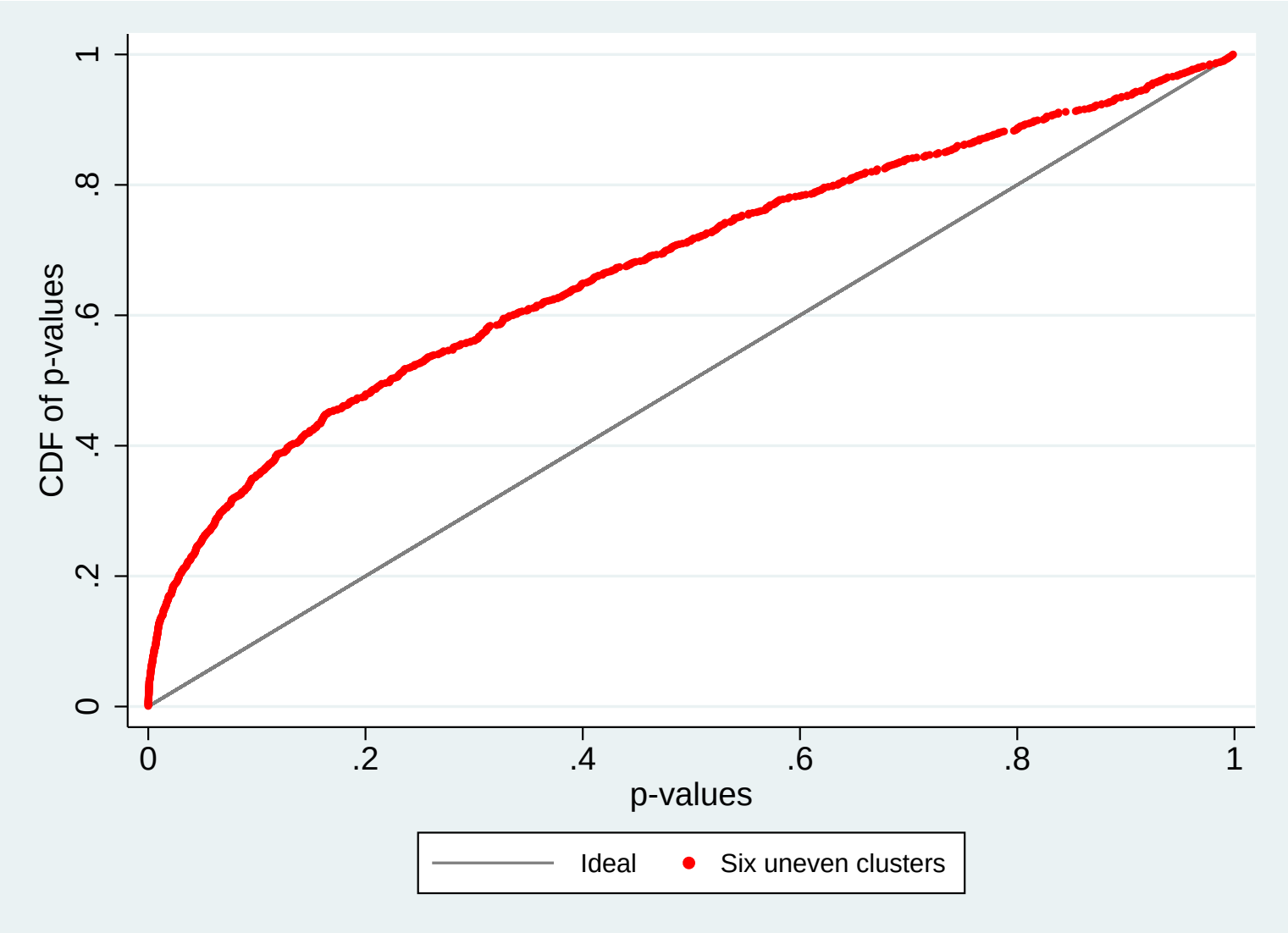
- The p-value for the the test based on the original sample statistic w equals the proportion of times that $|w| > |w^*_b|$.

What happens with six clusters



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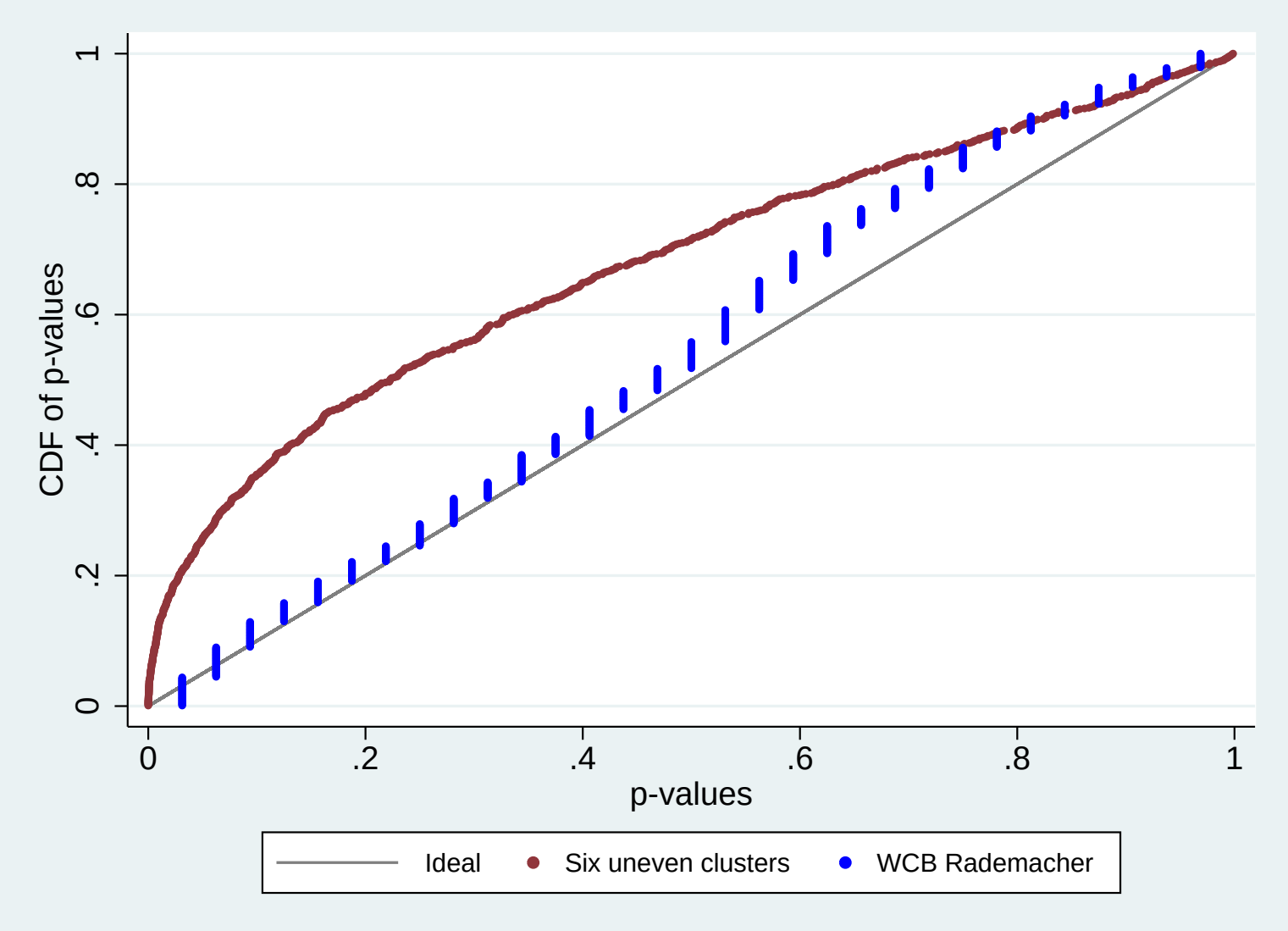
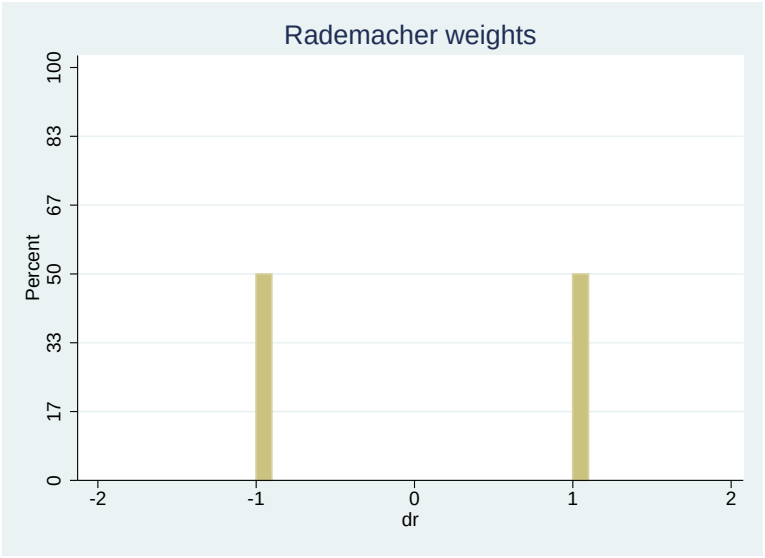


So-called Rademacher and Webb weights



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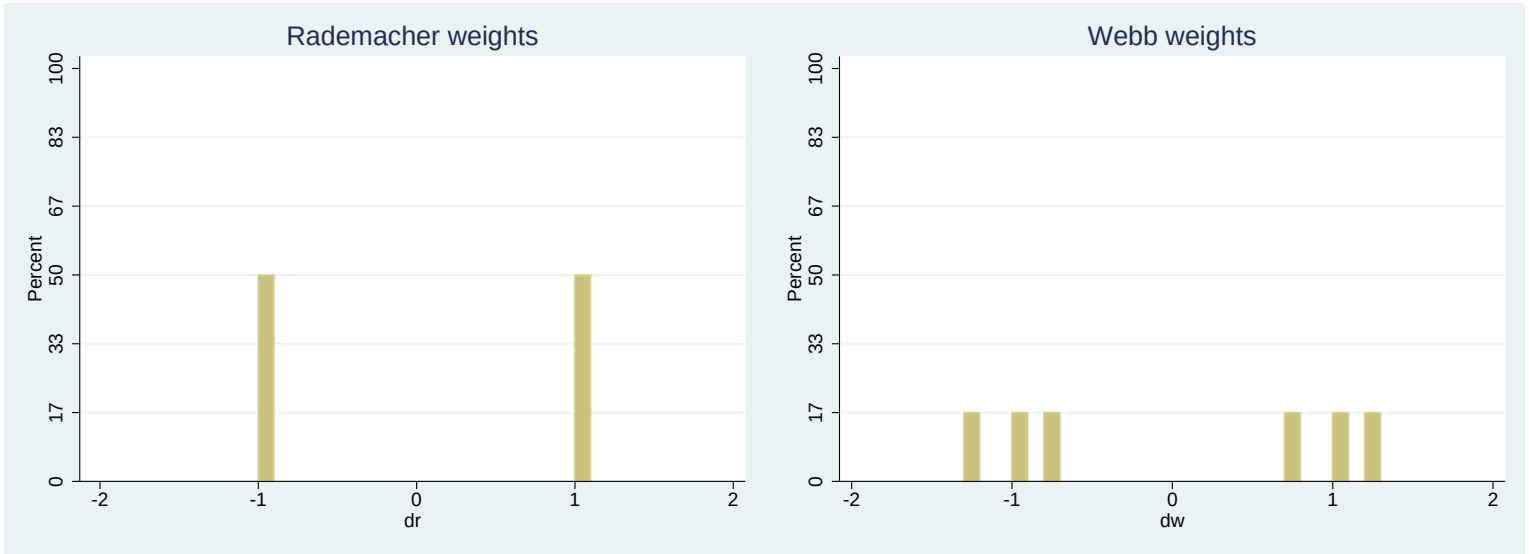

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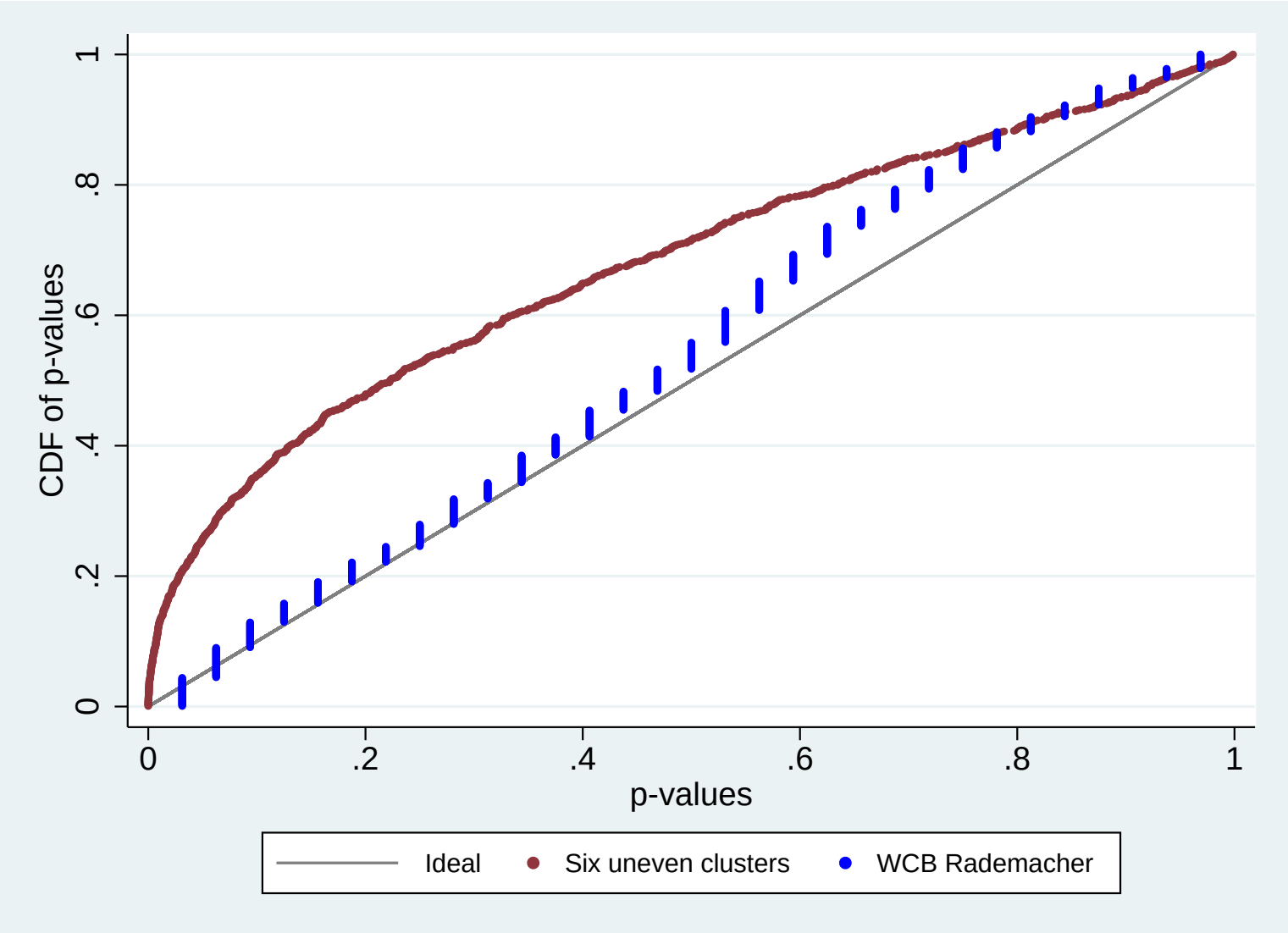

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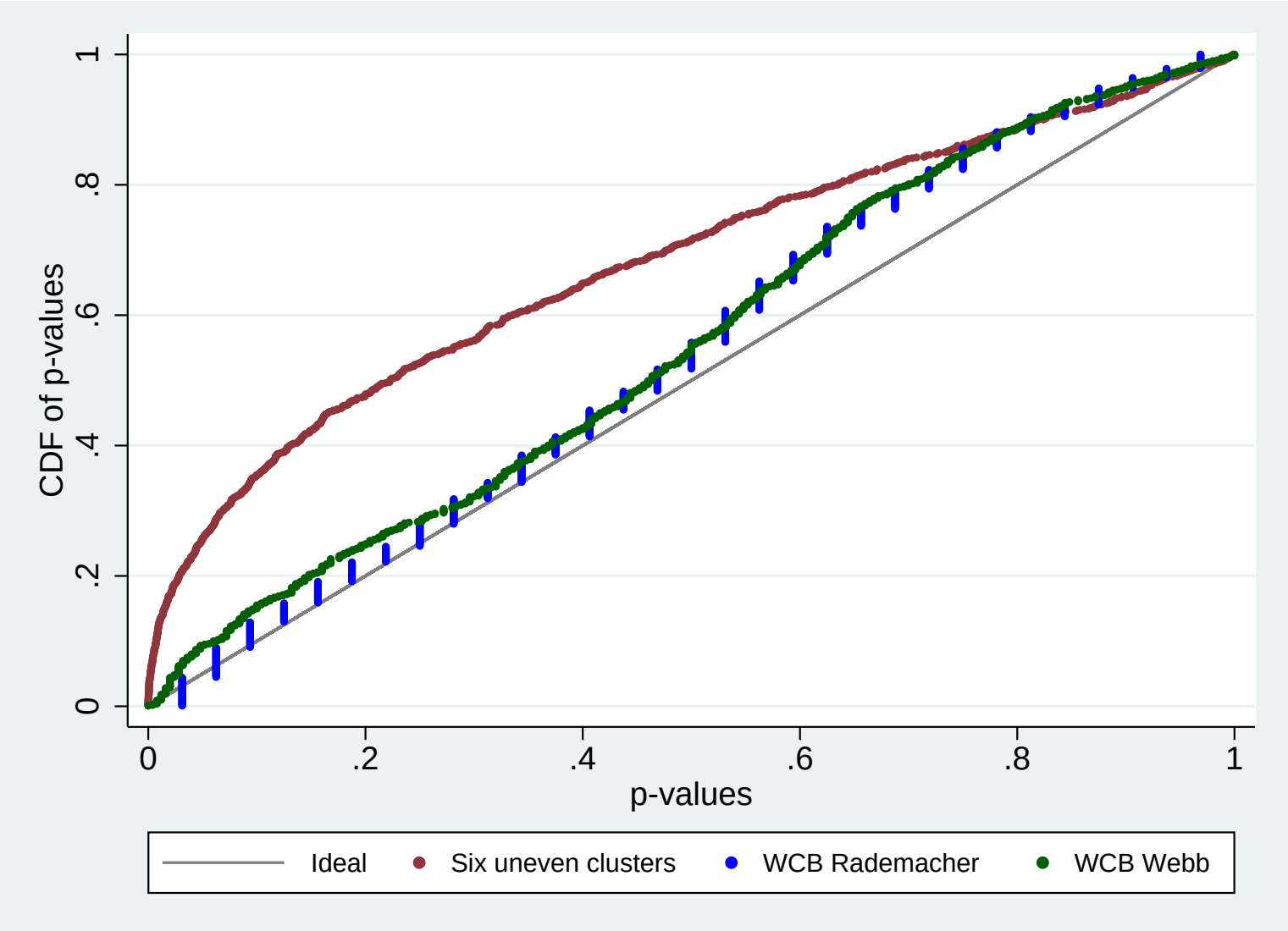

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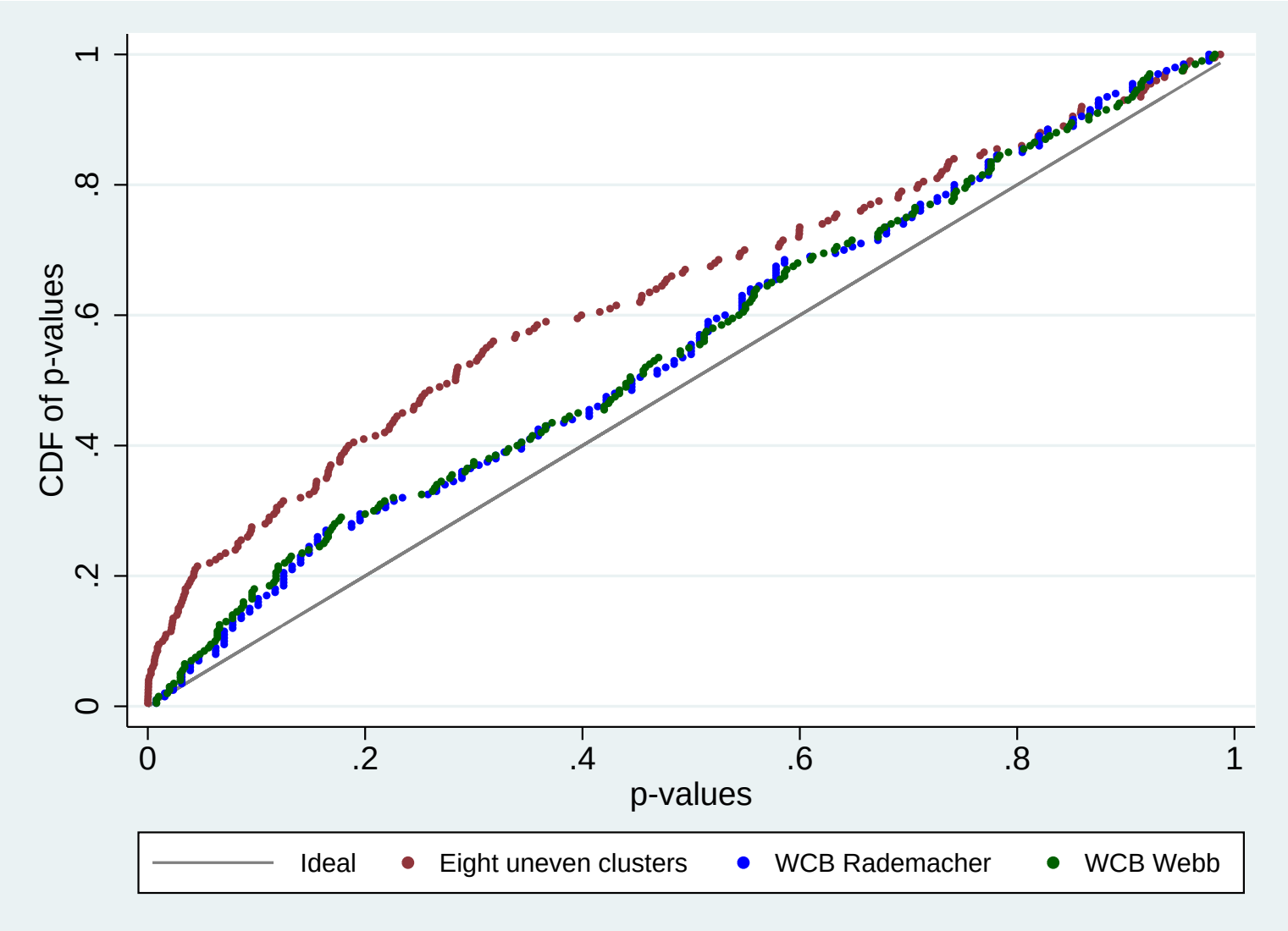
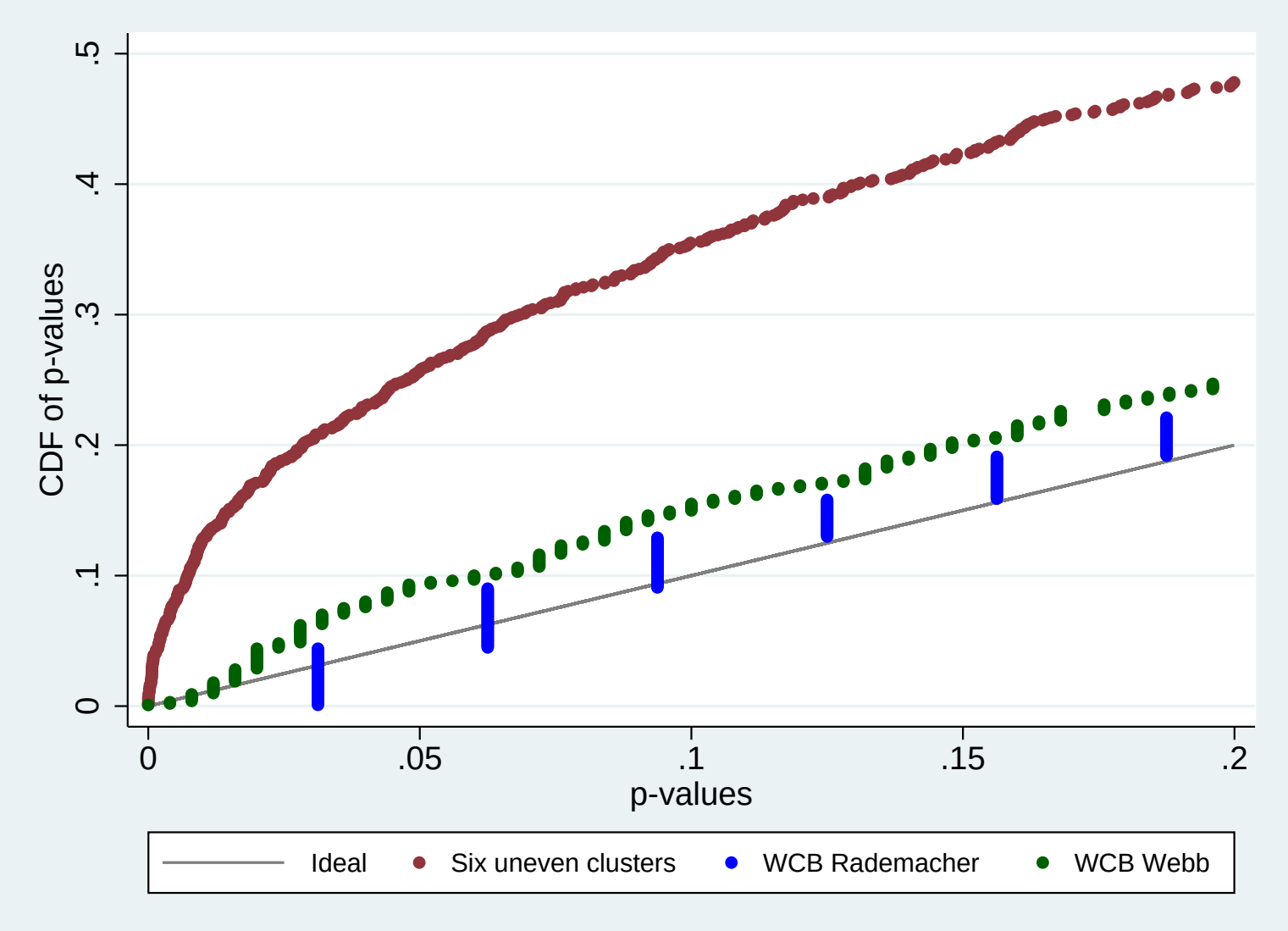


What happens with six clusters (zoom)



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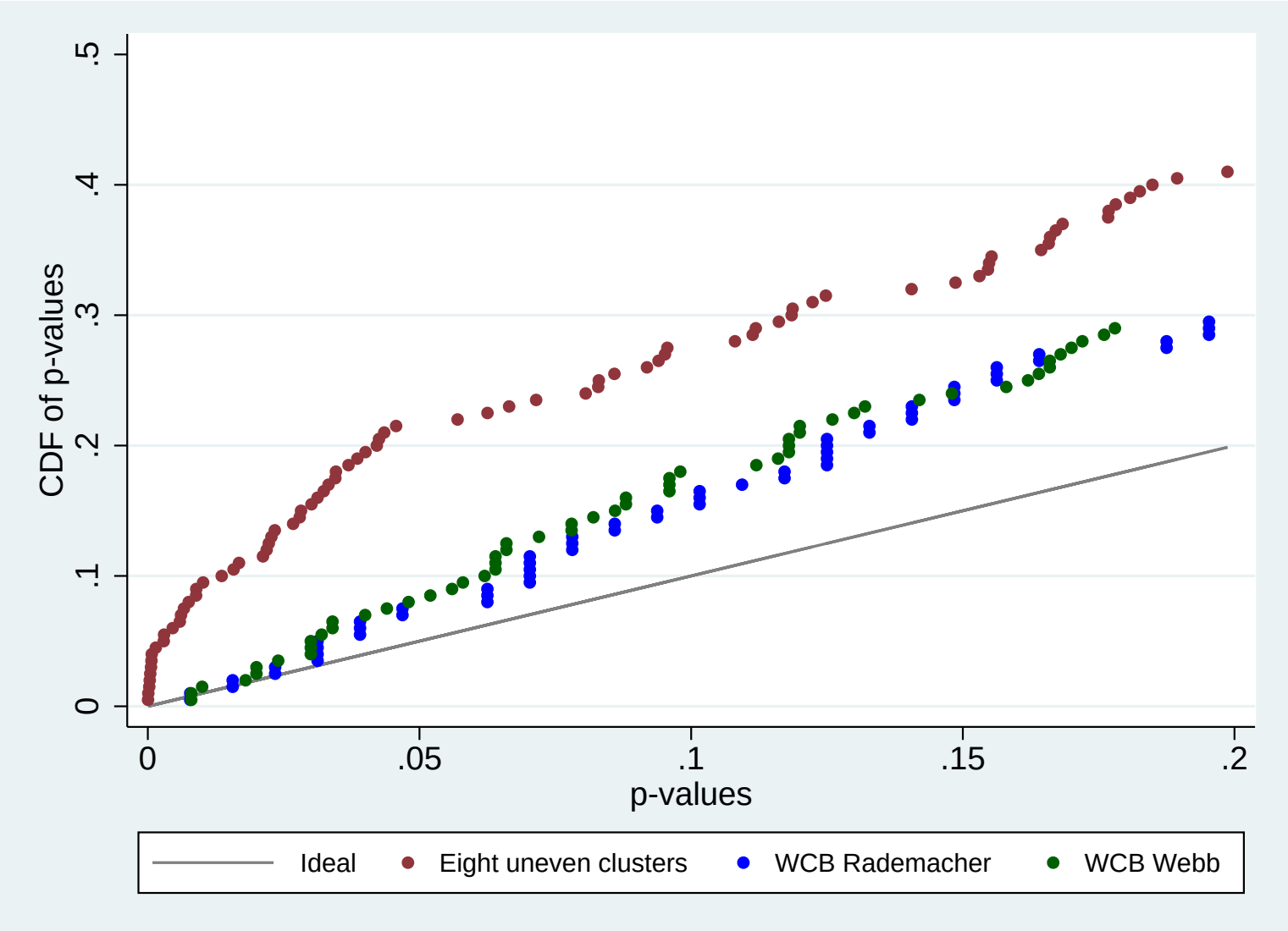

Activity 2