ECON 626: Applied Microeconomics

Lecture 7:

Power and Clustering

Professors: Pamela Jakiela and Owen Ozier

Department of Economics
University of Maryland, College Park

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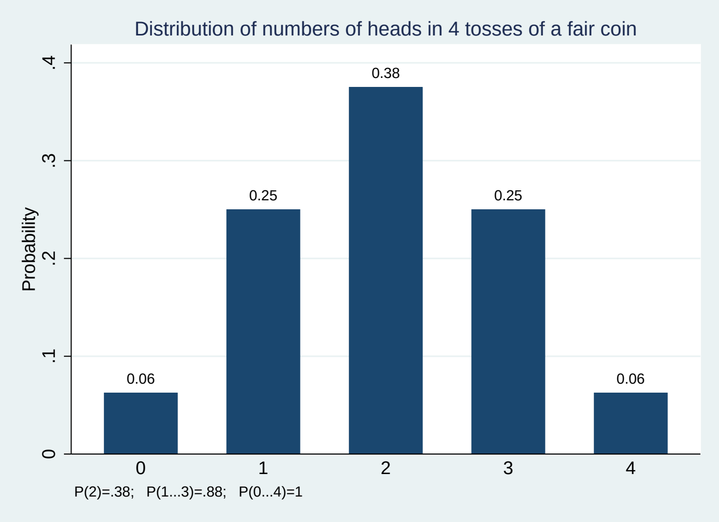
- We don't want to reject the null when it is true, though;
  How much accidental rejection would each possible cutoff give us?

Distribution of possible results



Types of error

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	"Reject Null," Find an effect!	"Fail to Reject Null," Conclude no effect.
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Power depends on anticipated effect size; we typically want power $\geq 80\%$.

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- What about 20 coin tosses?

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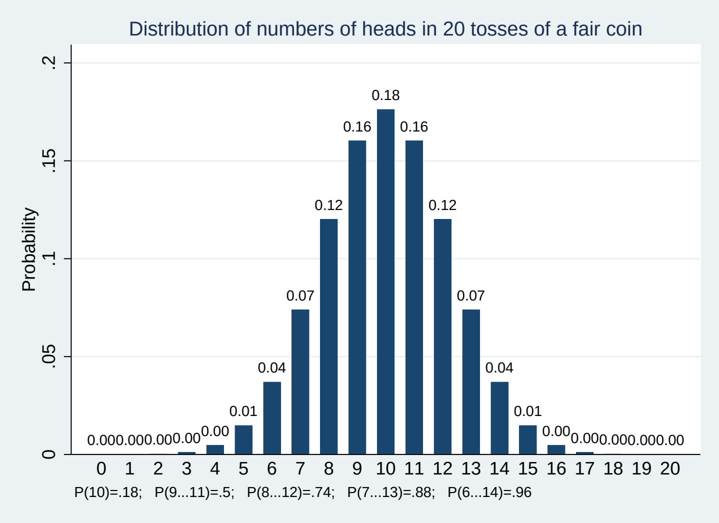
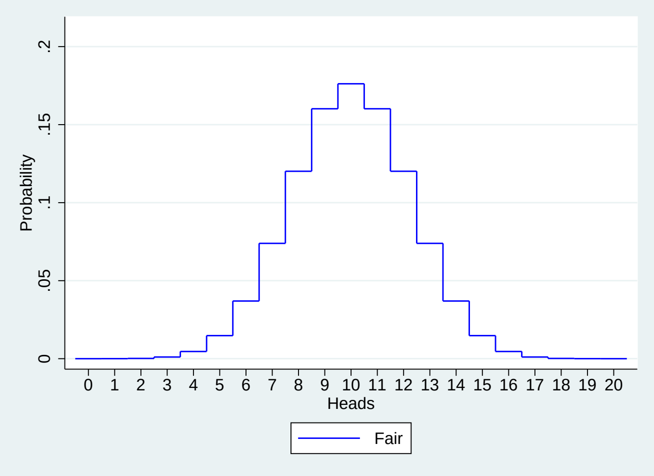


Power with 20 tosses



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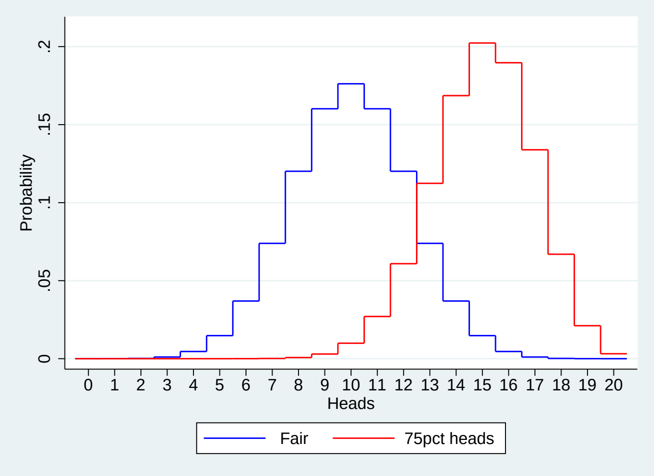

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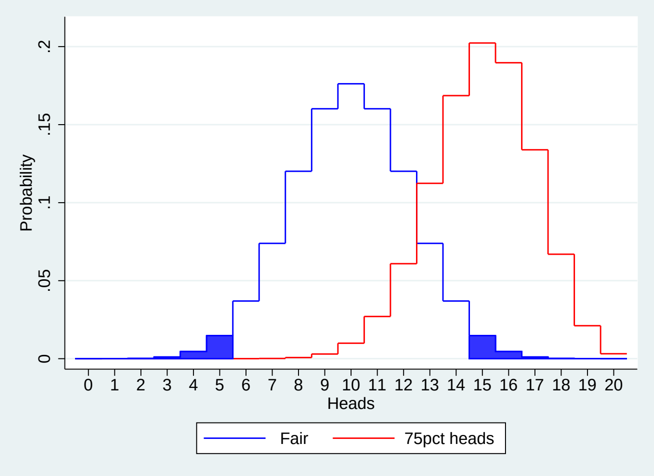

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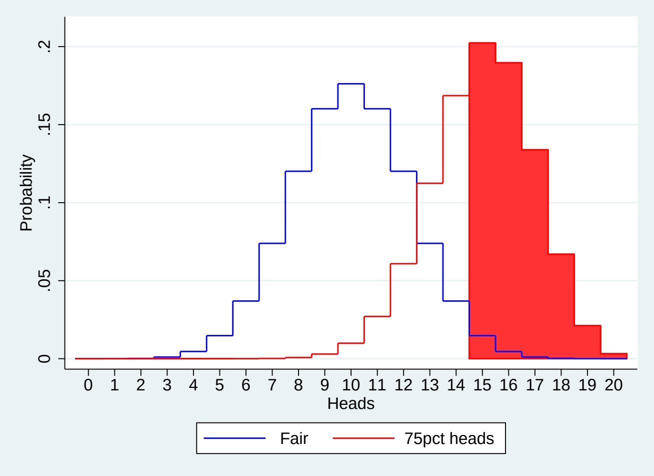
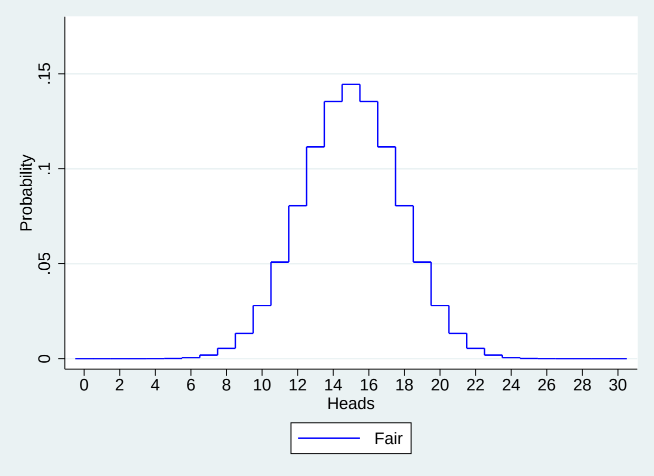


Power: about 0.62

Power with 30 tosses



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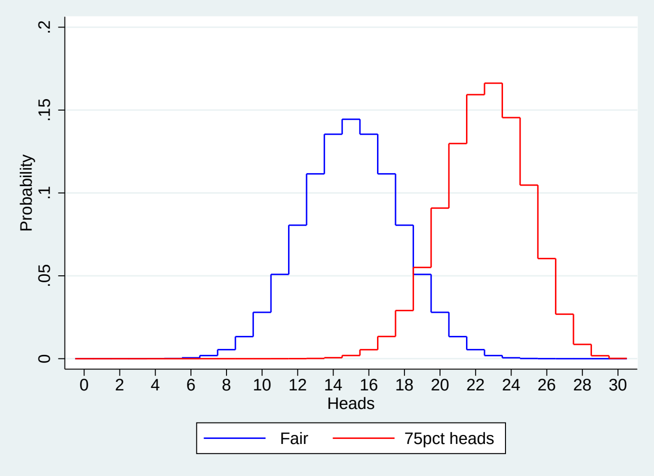

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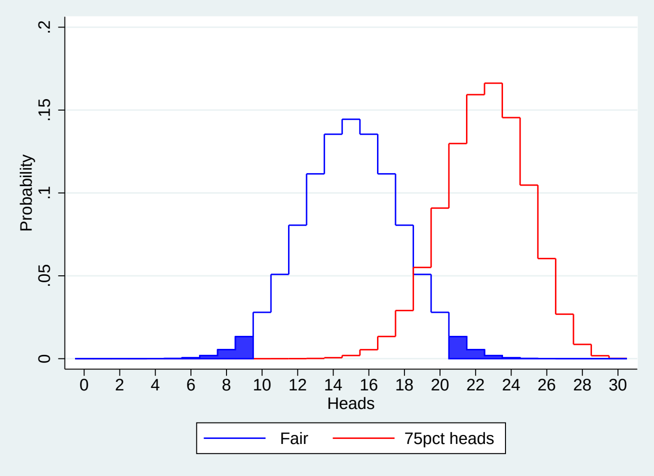

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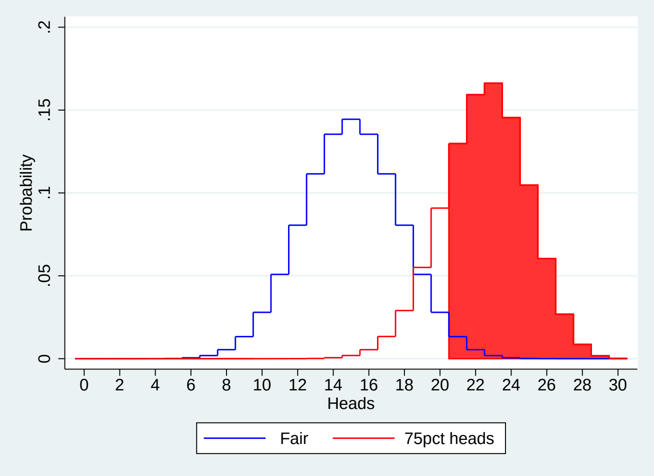
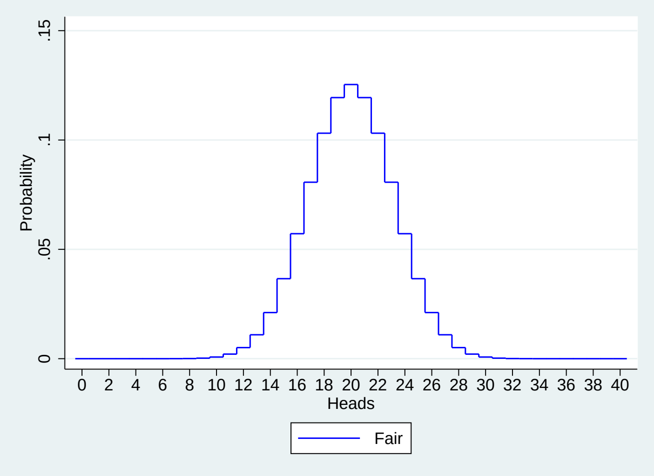


Power: about 0.80

Power with 40 tosses



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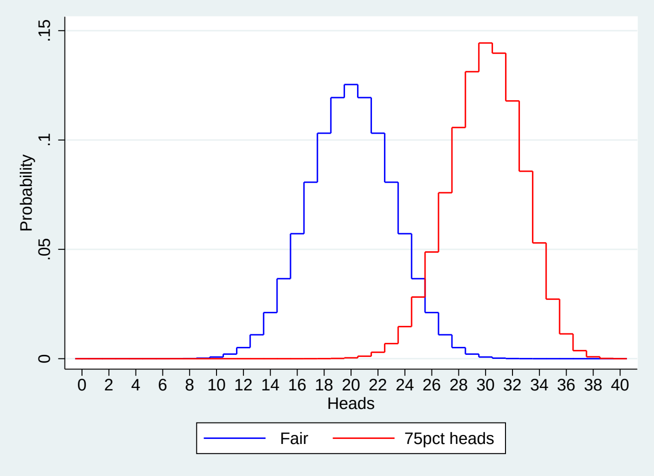

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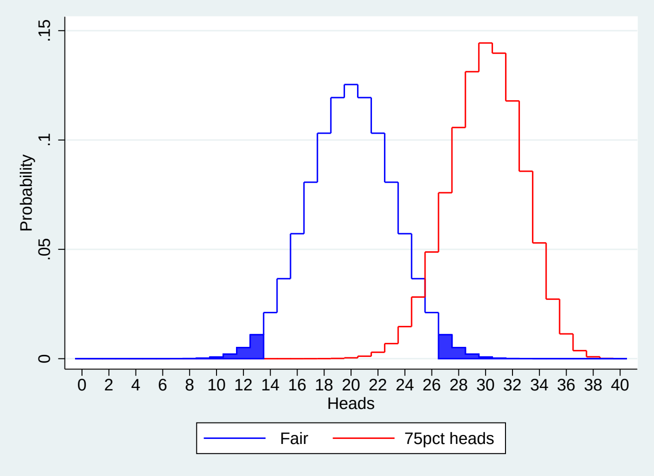

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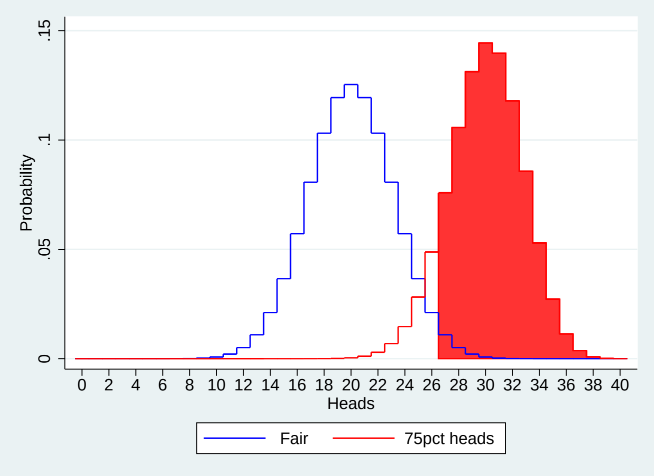
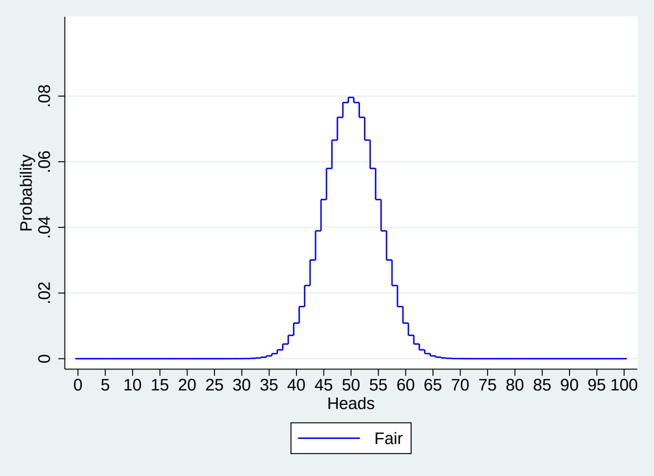


Power: about 0.90

Power with 100 tosses



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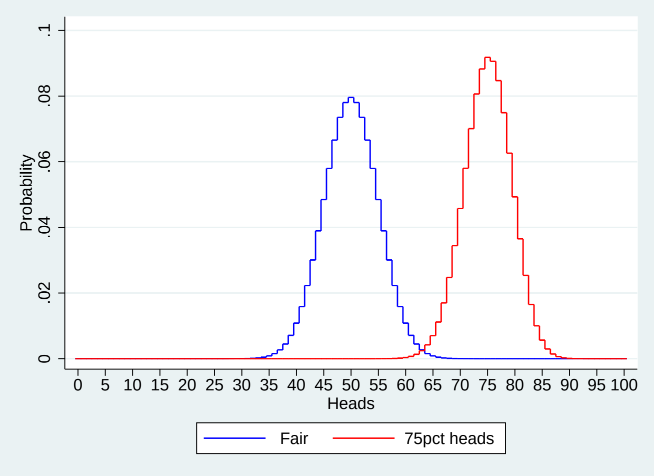

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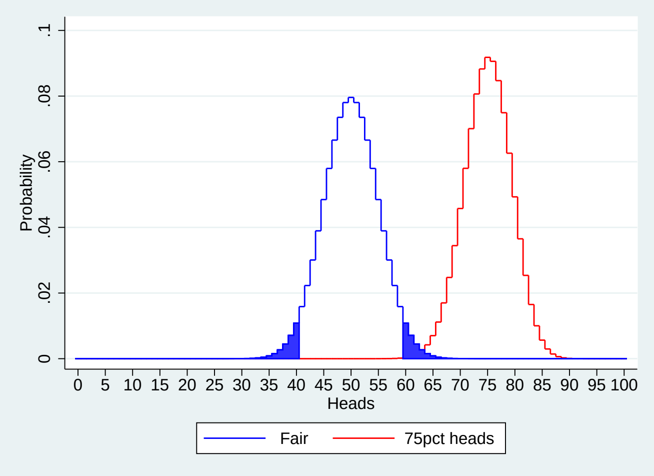

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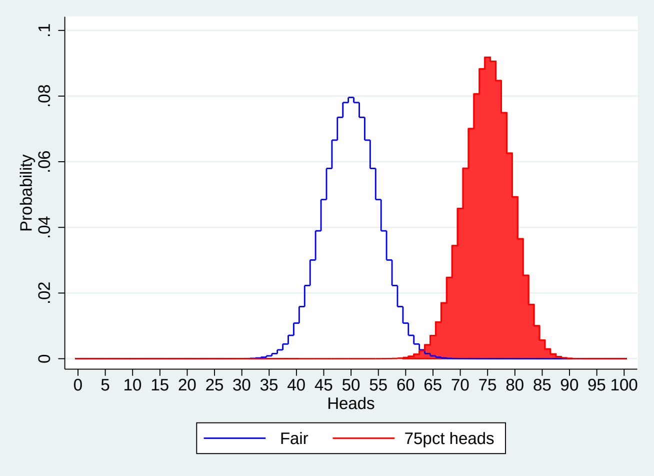
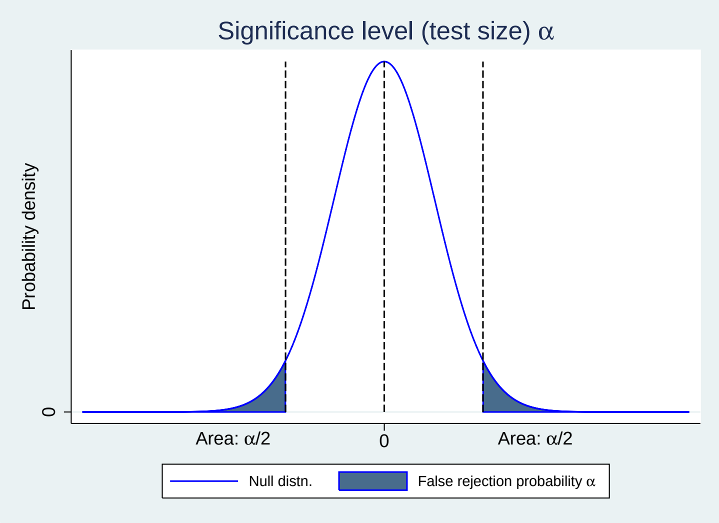
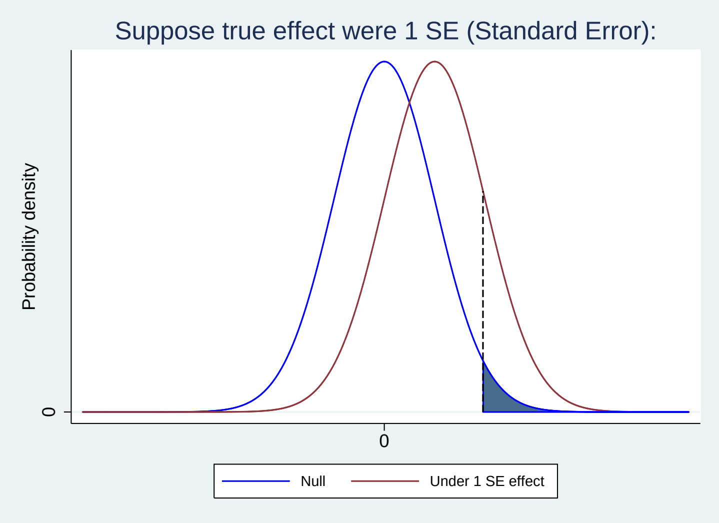


Power: about 0.9997

Rejecting H_0 in critical region



Under an alternative:



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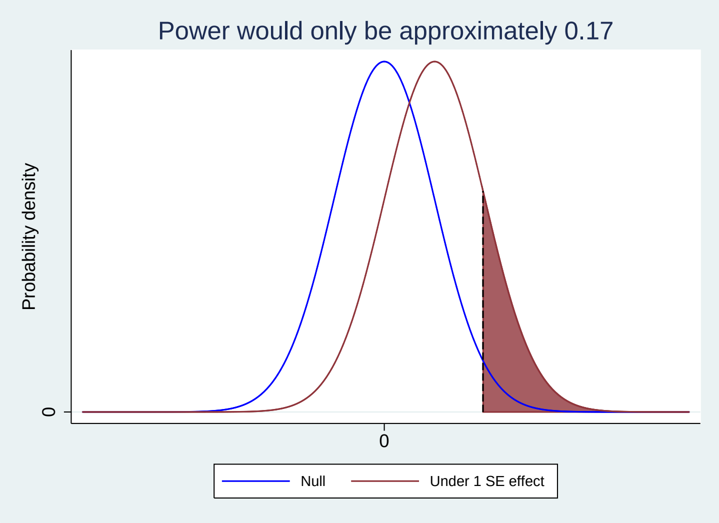

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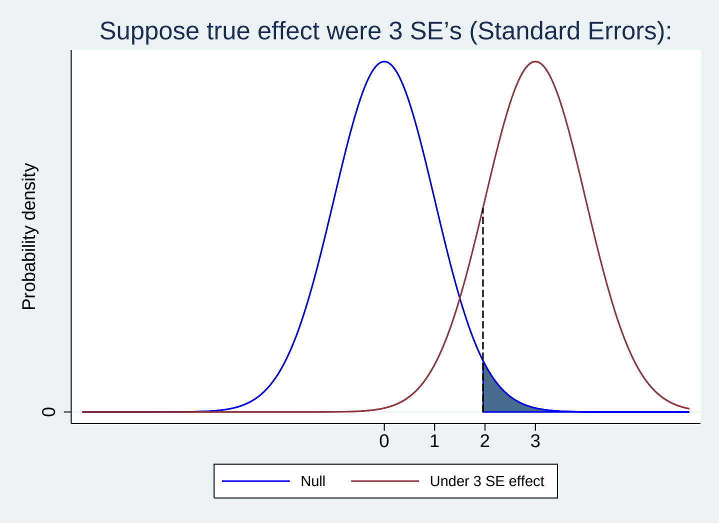

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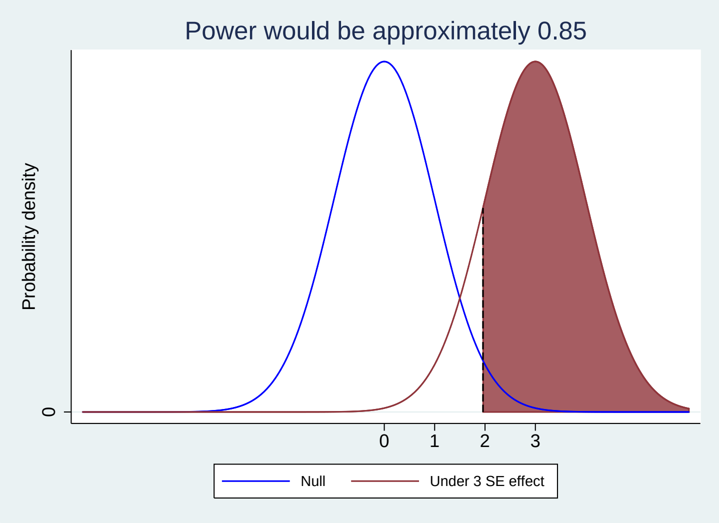
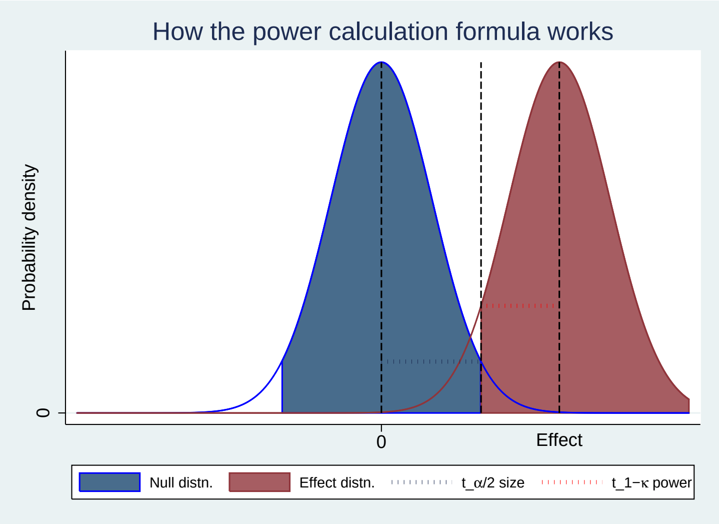


Power calculation, visually



Note: see the related figure in the *Toolkit* paper.

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$Effect > (t_{1-\kappa} + t_{\alpha/2}) SE(\hat{\beta})$ Notation: $t_{1-p} = p^{th}$ percentile of the t dist'n.

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- Consider standardized effect sizes in terms of standard deviations
- Draw on existing data: What is available that could inform your project?

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- **Classes within a school** are assigned to treatment or comparison; we observe outcomes at the level of the individual pupil
- **Households** are assigned to treatment or comparison; we observe outcomes at the level of the individual family member
- **Sub-district locations** are assigned to treatment or comparison; we observe outcomes at the level of the individual road
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What does this do?
It depends on how much variation is explained by the group each individual is in.

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This is the formula from before, with $P = 1/2$:

$$\sqrt{\frac{1}{P(1-P)}}\sqrt{\frac{\sigma^2}{N}}$$

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Now suppose $y_i = \beta t_i + \epsilon_i$, but $\epsilon_i = \nu_g + \eta_{ig}$ for groups g of fixed size n_g. We still compare the means of those with $t_i = 1$ to those with $t_i = 0$.

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$$\rho_\epsilon = \frac{\sigma_\nu^2}{\sigma_\nu^2 + \sigma_\eta^2} = \frac{\sigma_\nu^2}{\sigma_\epsilon^2}$$

Two other ways of writing this will be convenient:

$$\sigma_\nu^2 = \rho_\epsilon \sigma_\epsilon^2$$

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Two other ways of writing this will be convenient:

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What happens to the variance of the estimator?

As before,

$$\hat{\beta} = \beta + \frac{1}{N/2} \sum_{T} \epsilon_i - \frac{1}{N/2} \sum_{C} \epsilon_i$$

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$$\begin{aligned} Var(\hat{\beta}) =& \frac{1}{N/(2n_g)}\sigma_\nu^2 + \frac{1}{N/2}\sigma_\eta^2 + \frac{1}{N/(2n_g)}\sigma_\nu^2 + \frac{1}{N/2}\sigma_\eta^2 \\ =& \frac{4n_g}{N}\sigma_\nu^2 + \frac{4}{N}\sigma_\eta^2 \end{aligned}$$

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Var(\hat{\beta}) &= \frac{1}{N/(2n_g)}\sigma_\nu^2 + \frac{1}{N/2}\sigma_\eta^2 + \frac{1}{N/(2n_g)}\sigma_\nu^2 + \frac{1}{N/2}\sigma_\eta^2 \\
&= \frac{4n_g}{N}\sigma_\nu^2 + \frac{4}{N}\sigma_\eta^2 \\
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Var(\hat{\beta}) =& \frac{1}{N/(2n_g)}\sigma_\nu^2 + \frac{1}{N/2}\sigma_\eta^2 + \frac{1}{N/(2n_g)}\sigma_\nu^2 + \frac{1}{N/2}\sigma_\eta^2 \\
=& \frac{4n_g}{N}\sigma_\nu^2 + \frac{4}{N}\sigma_\eta^2 \\
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$$SE(\hat{\beta}) = \sqrt{4}\sqrt{\frac{\sigma_\epsilon^2}{N}}\sqrt{(n_g-1)\rho_\epsilon + 1} = \sqrt{\frac{1}{P(1-P)}}\sqrt{\frac{\sigma^2}{N}}\sqrt{(n_g-1)\rho_\epsilon + 1}$$

The formula

Scale the effective standard error by:

$$\textit{Design Effect (``Moulton factor'')} = \sqrt{1 + (n_{groupsize} - 1)\rho}$$

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Estimation example: clustered standard errors

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$$\hat{\Omega}_{cl} = (X'X)^{-1} \left(\sum_g X_g' \hat{\Psi}_g X_g \right) (X'X)^{-1}$$

where

$$\hat{\Psi}_g = a\hat{e}_g \hat{e}_g' = a \begin{bmatrix} \hat{e}_{1g}^2 & \hat{e}_{1g}\hat{e}_{2g} & \dots & \hat{e}_{1g}\hat{e}_{n_g g} \\ \hat{e}_{2g}\hat{e}_{1g} & \hat{e}_{2g}^2 & \dots & \hat{e}_{2g}\hat{e}_{n_g g} \\ \dots & \dots & \dots & \dots \\ \hat{e}_{n_g g}\hat{e}_{1g} & \hat{e}_{n_g g}\hat{e}_{2g} & \dots & \hat{e}_{n_g g}^2 \end{bmatrix}$$

Estimation example: clustered standard errors

But remember, in the simplest case, X_g' is either:

$$\begin{bmatrix} 1 & 1 & ... & 1 \\ 1 & 1 & ... & 1 \end{bmatrix} \textit{ or } \begin{bmatrix} 0 & 0 & ... & 0 \\ 1 & 1 & ... & 1 \end{bmatrix}$$

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Count the terms. diagonal:

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Diagonal terms have expectation

Estimation example: clustered standard errors

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- Consider what might be reasonable assumptions
- Draw on existing data (again):
  What is available that could inform your project?

Intra-class correlations we have known

Data source	ICC (ρ)
Madagascar Math + Language	0.5
Busia, Kenya Math + Language	0.22
Udaipur, India Math + Language	0.23
Mumbai, India Math + Language	0.29
Vadodara, India Math + Language	0.28
Busia, Kenya Math	0.62
Busia, Kenya Language	0.43
Busia, Kenya Science	0.35

Duflo, Glennerster, and Kremer (2006) Using Randomization in Development Economics Research: A Toolkit

Data source	ICC (ρ)
US Elementary Math, unconditional	0.22
US Elementary Math, rural only, unconditional	0.15
US Elementary Math, rural only, conditional on previous scores	0.12

Hedges & Hedberg (2007), Intraclass correlations for planning group randomized experiments in rural education.

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- *"A first comment is that, despite all the precision of these formulas, power calculations involve substantial guess work in practice."*