ECON 626: Applied Microeconomics

Lecture 7:

Power

Professors: Pamela Jakiela and Owen Ozier

Lecture 7, Part 1:

Power in Randomized Trials

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  This means predicting... the standard error.

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  Could fail to reject under any of these conditions:
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  - or (D) always.

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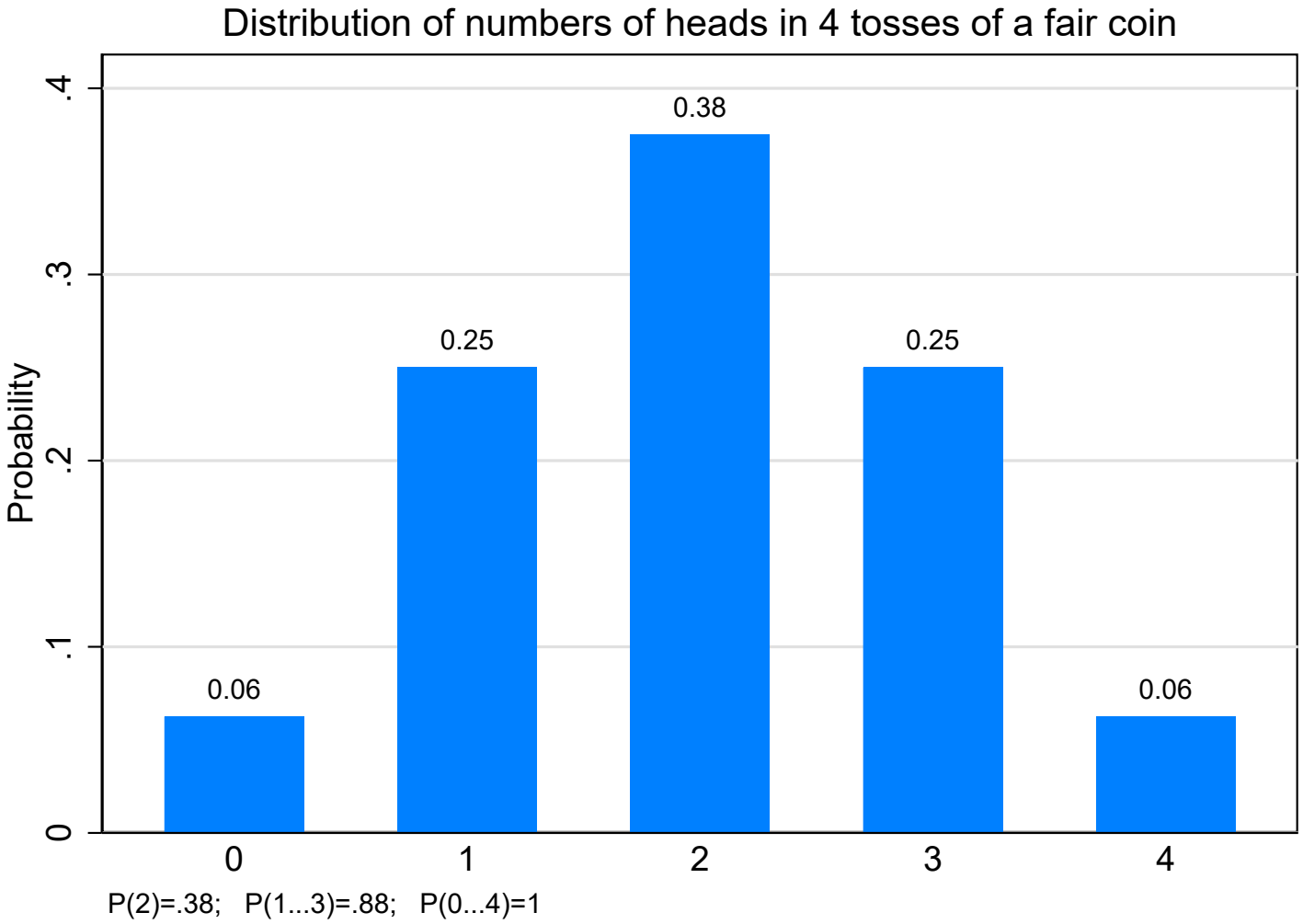
  - or (D) always.
- We don't want to reject the null when it is true, though;
  How much accidental rejection would each possible cutoff give us?

Distribution of possible results



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Power depends on anticipated effect size; we typically want power $\geq 80\%$.

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Calculating size and power with "reject null of fairness if HH or TT" rule:

	Probabilities	
	HH or TT "Reject Null," Find an effect!	HT or TH "Fail to Reject Null," No evidence of effect.
Unfair coin 1 ($p_H = 0.1$)		
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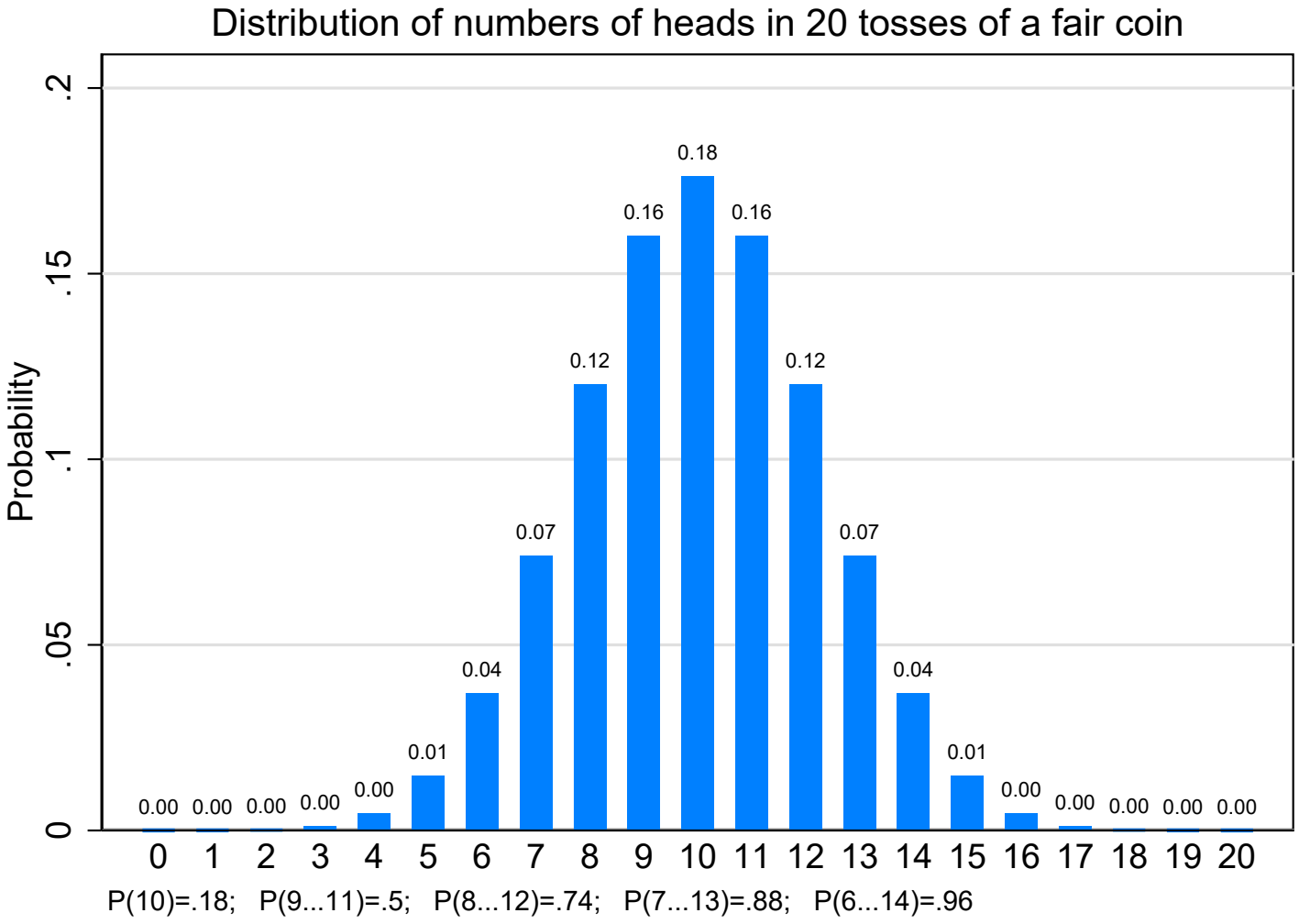
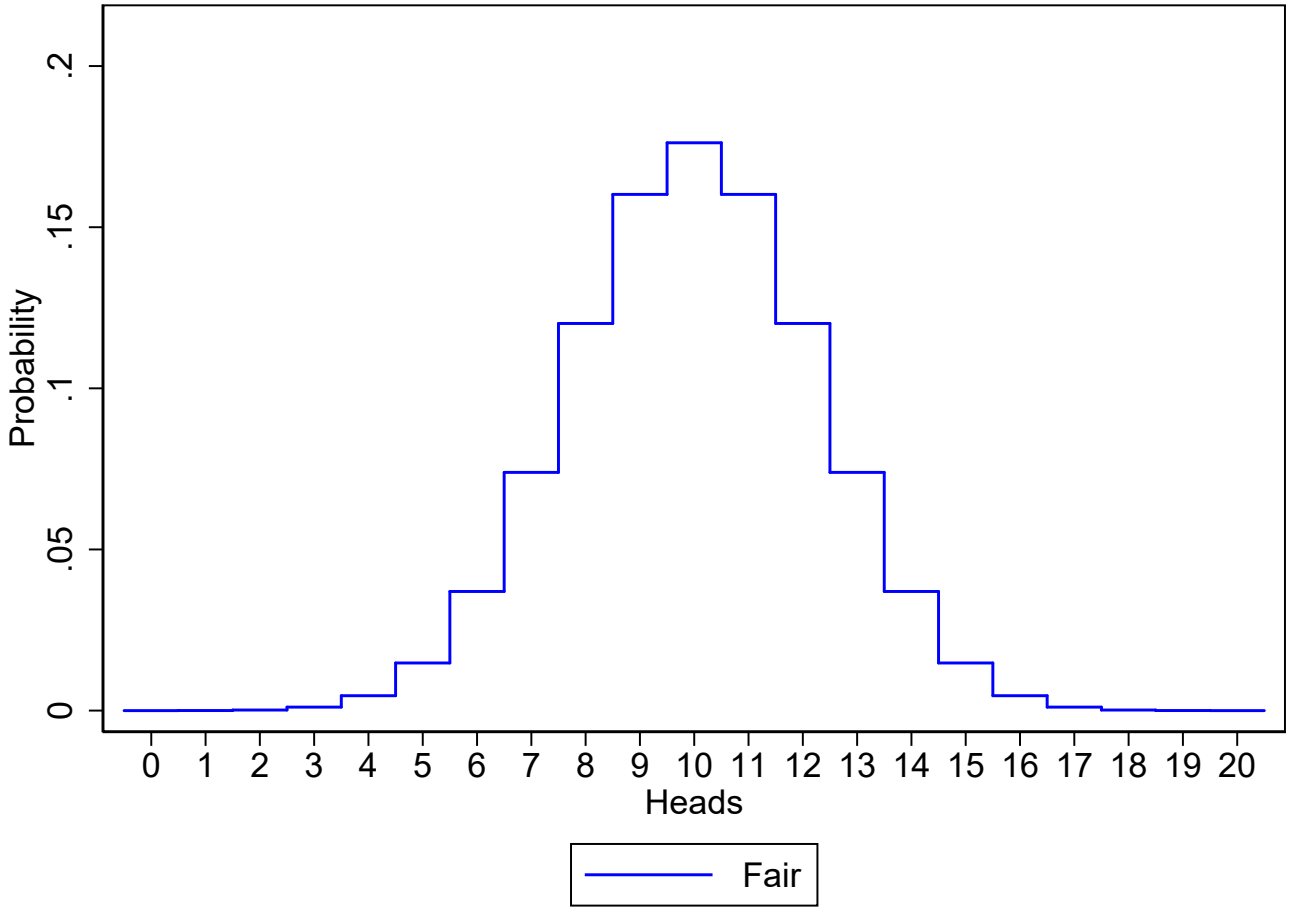
  ** (Except the "never reject, no matter what" rule. Not very useful.)*
- What about 20 coin tosses?

Distribution of possible results



Power with 20 tosses



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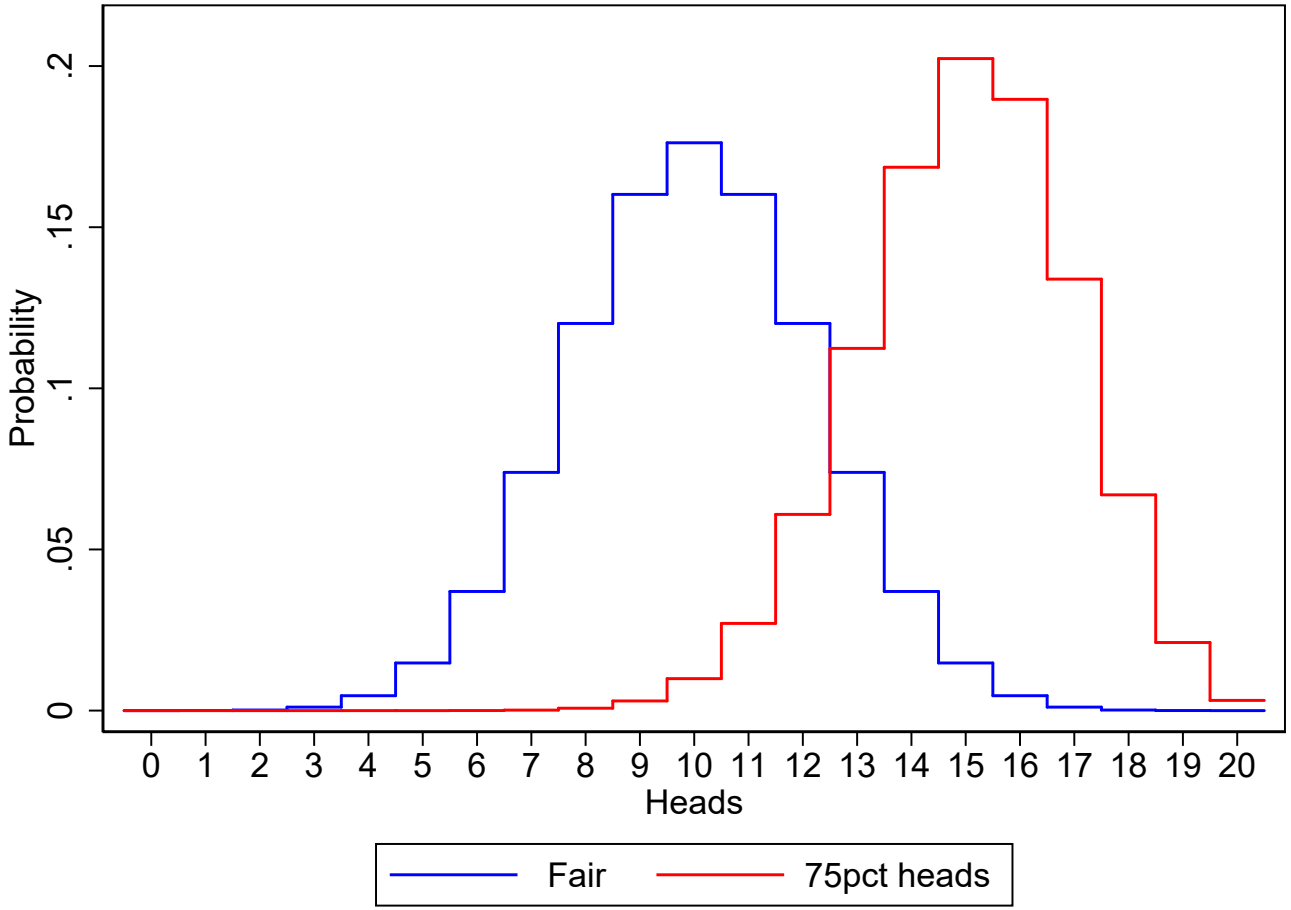

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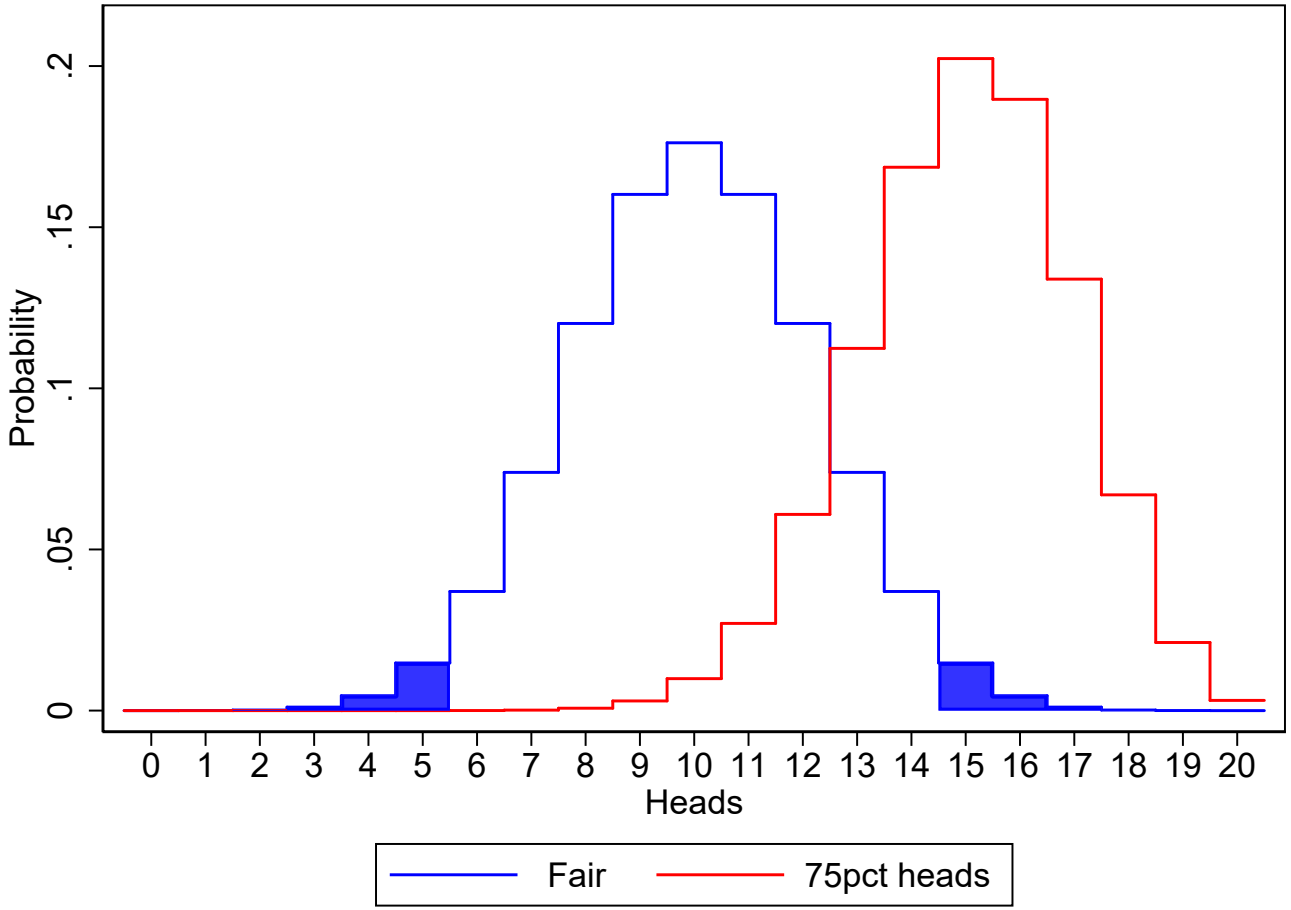

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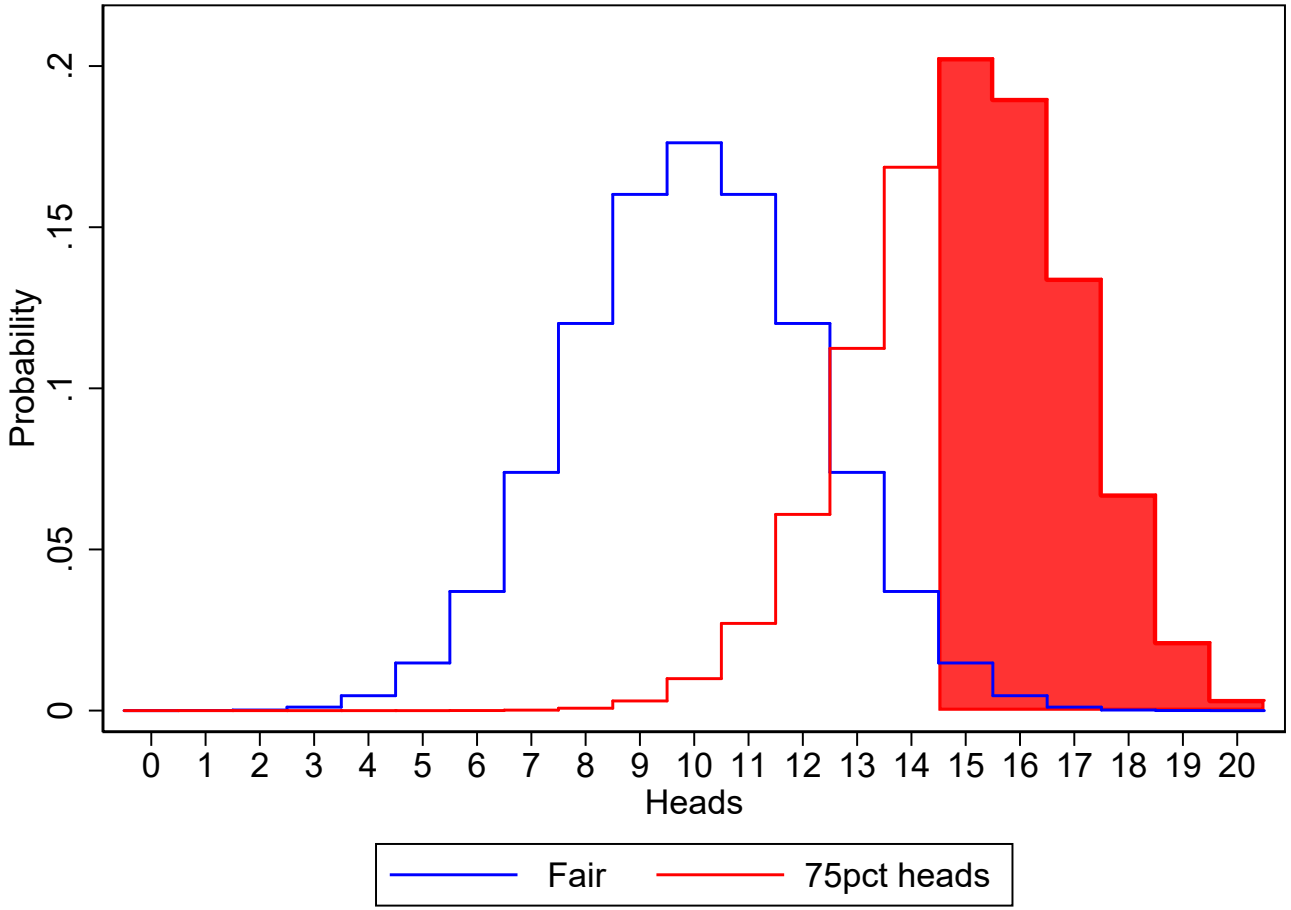
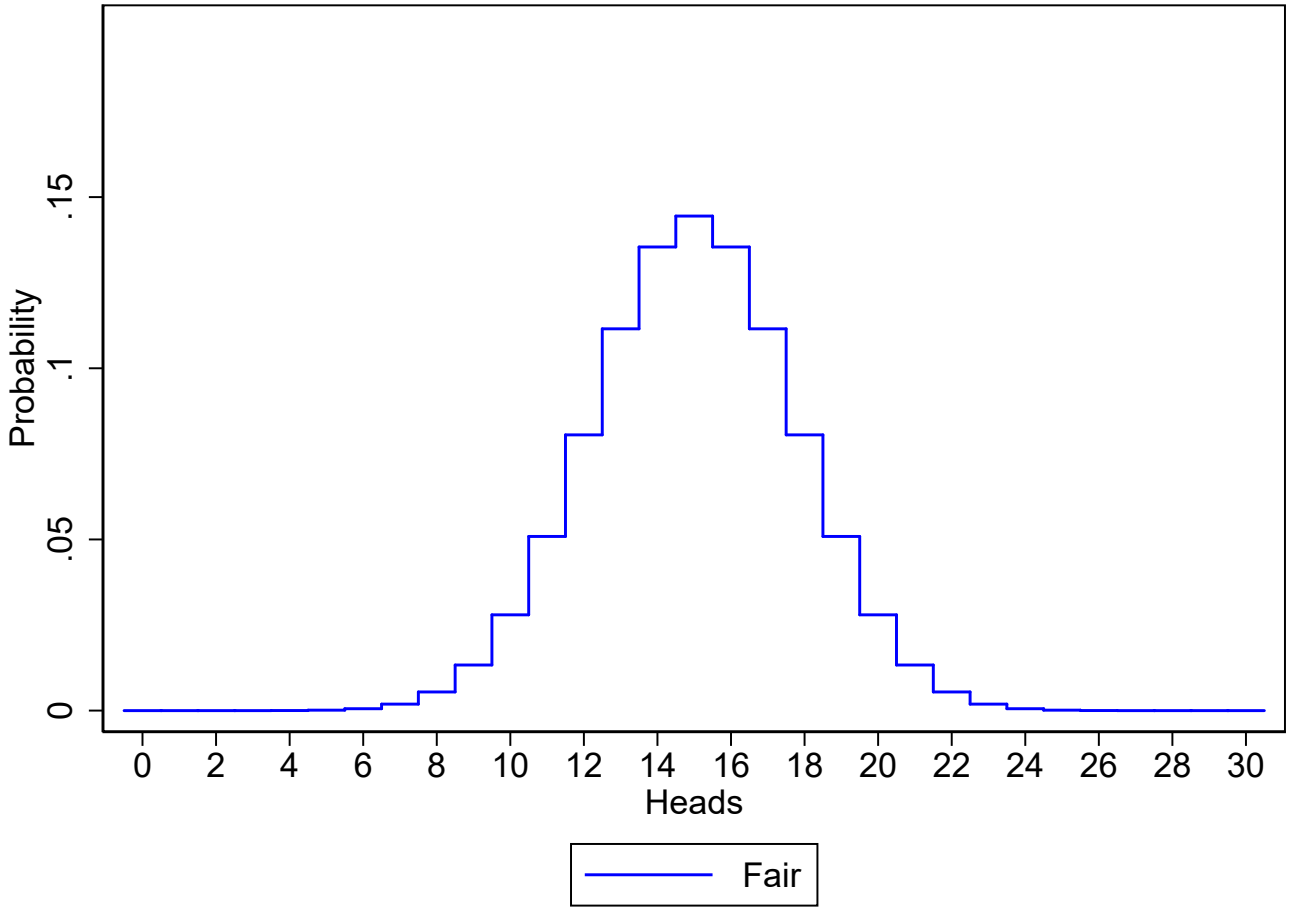


Power: about 0.62

Power with 30 tosses



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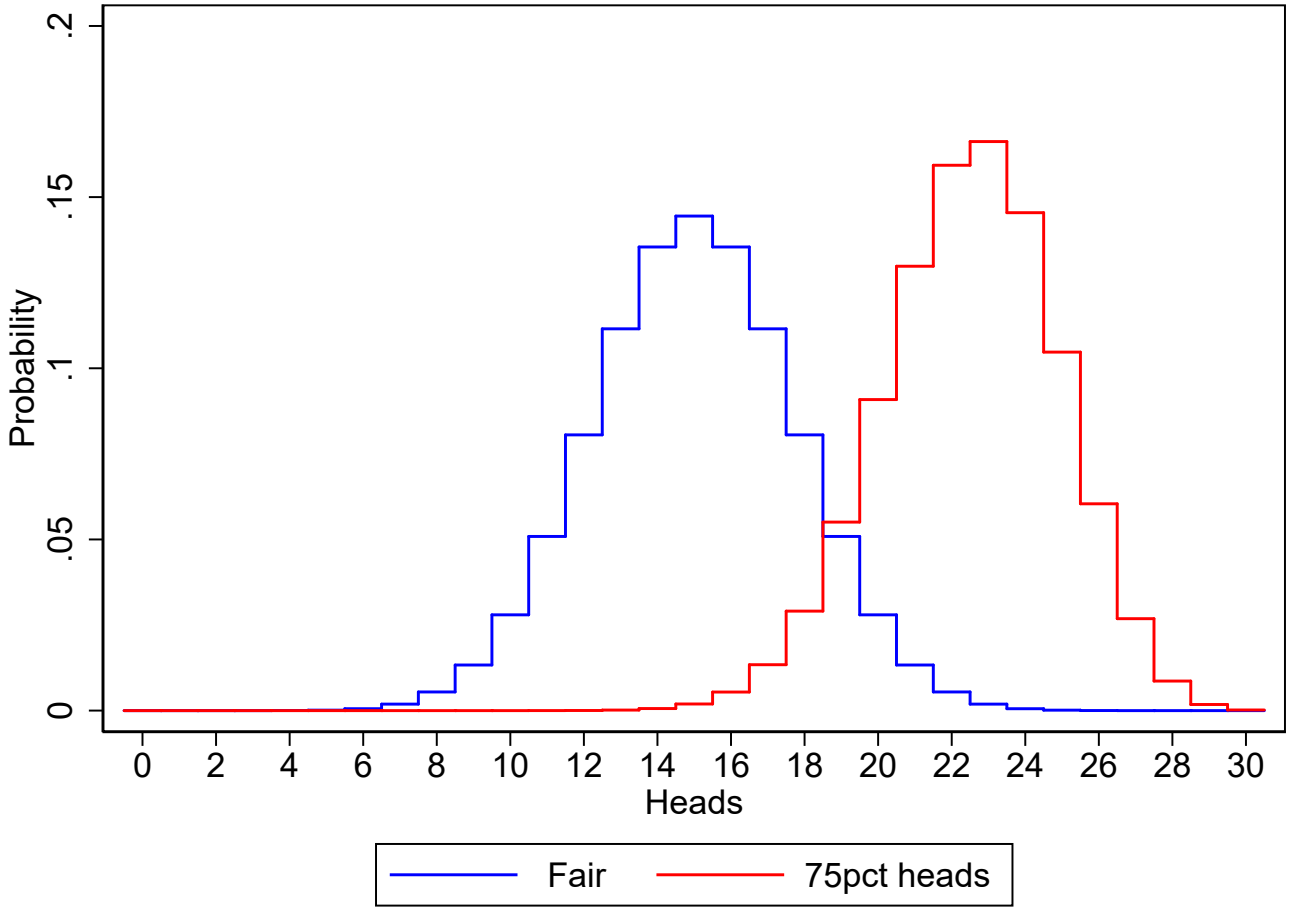

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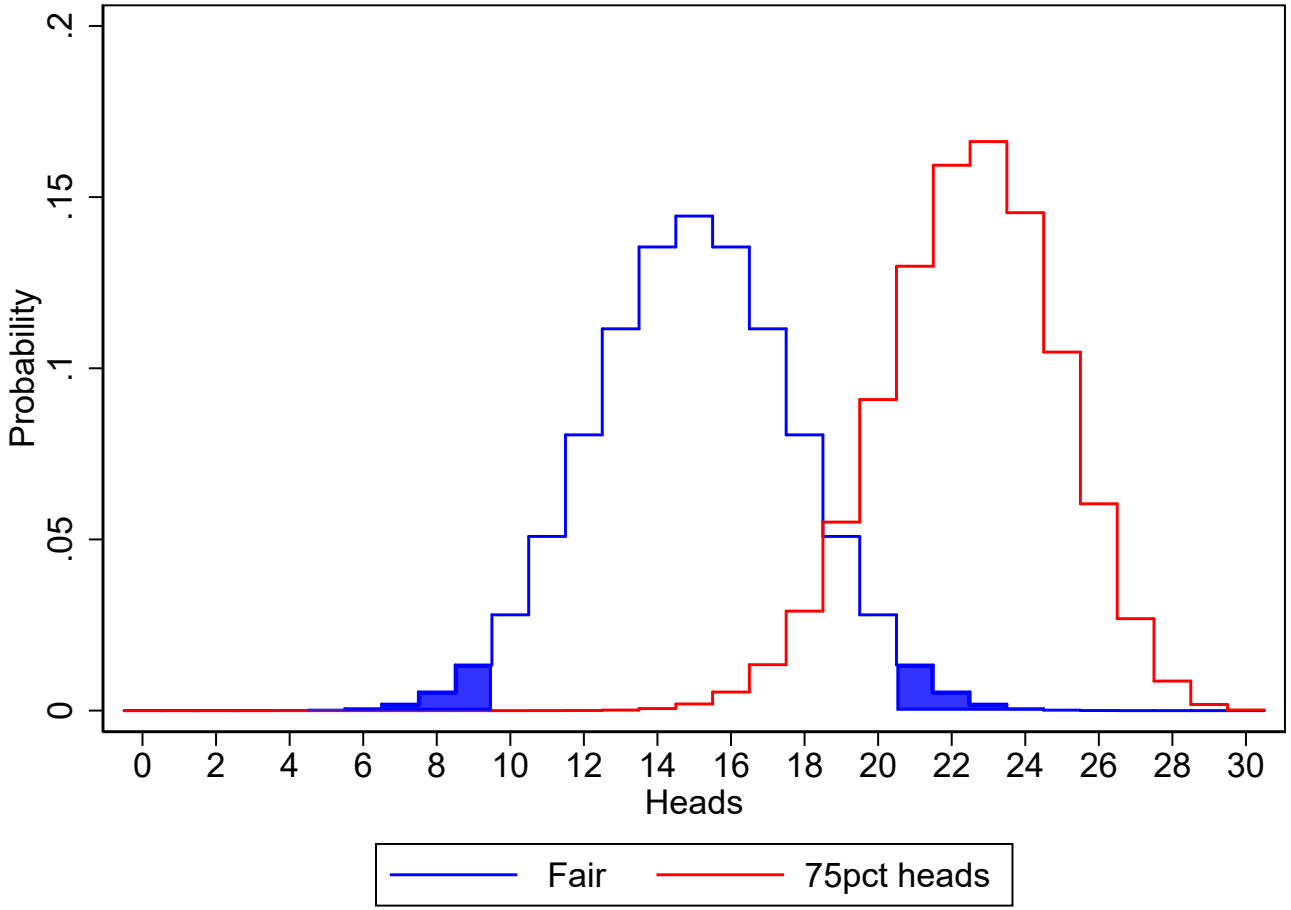

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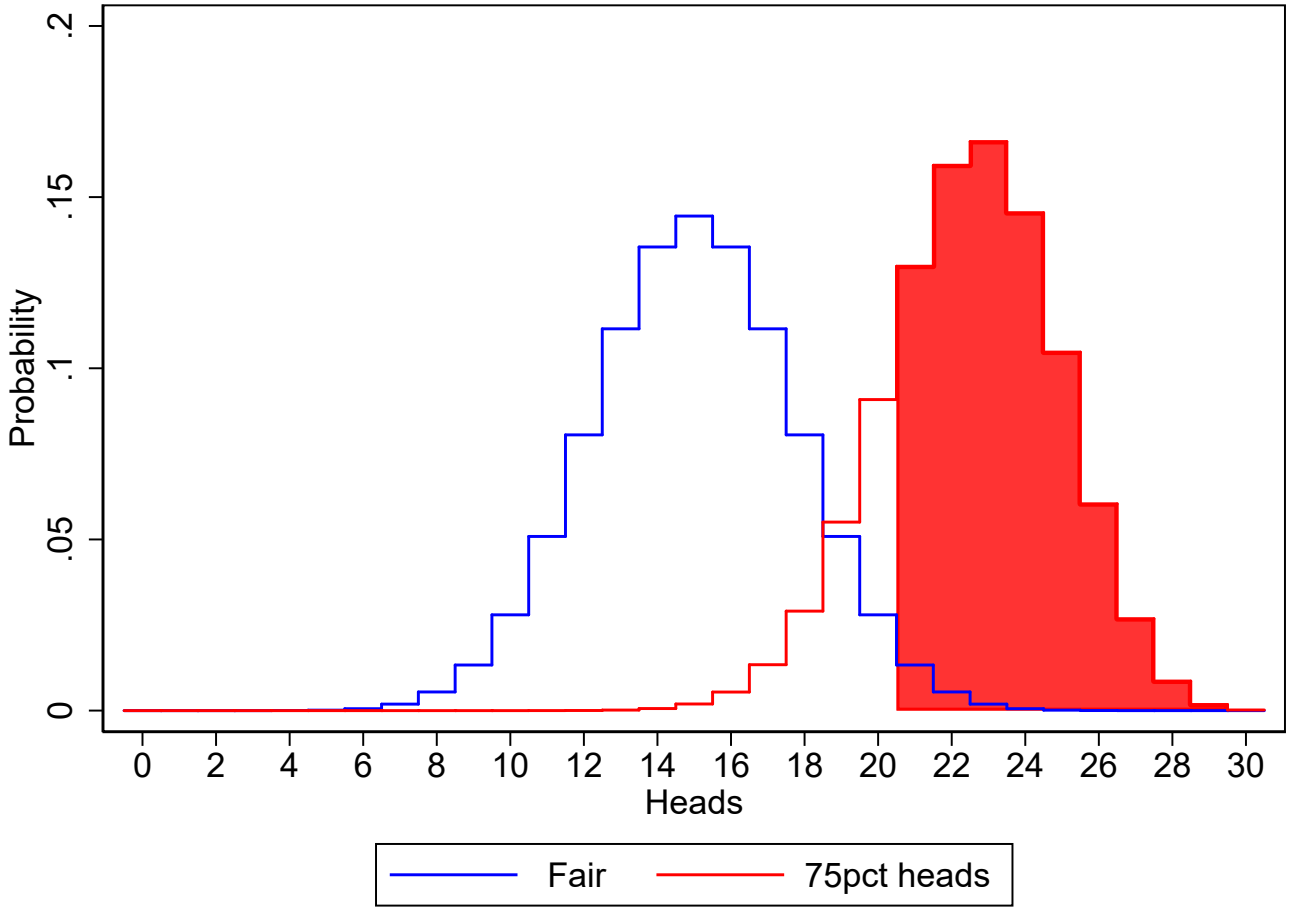
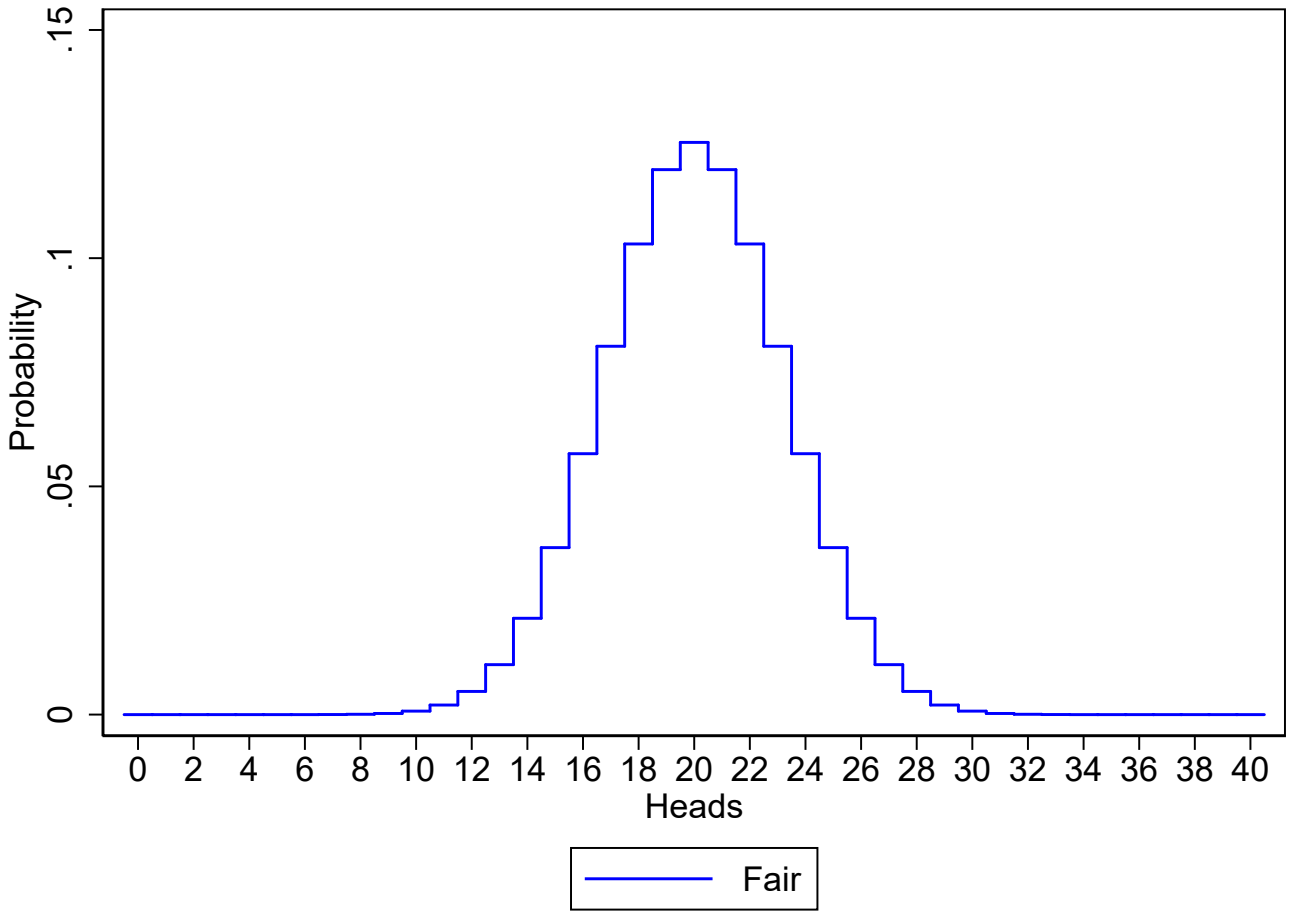


Power: about 0.80

Power with 40 tosses



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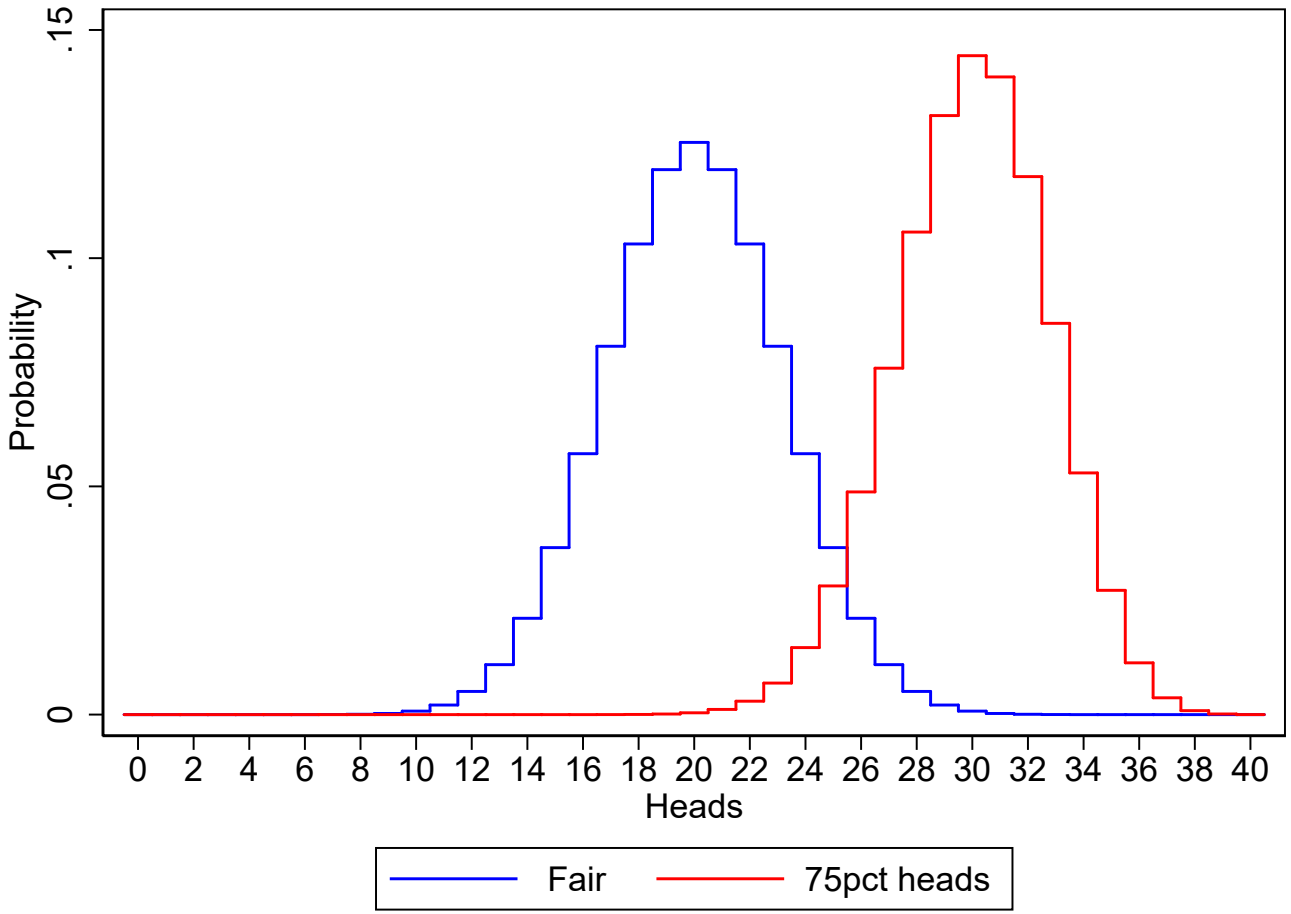

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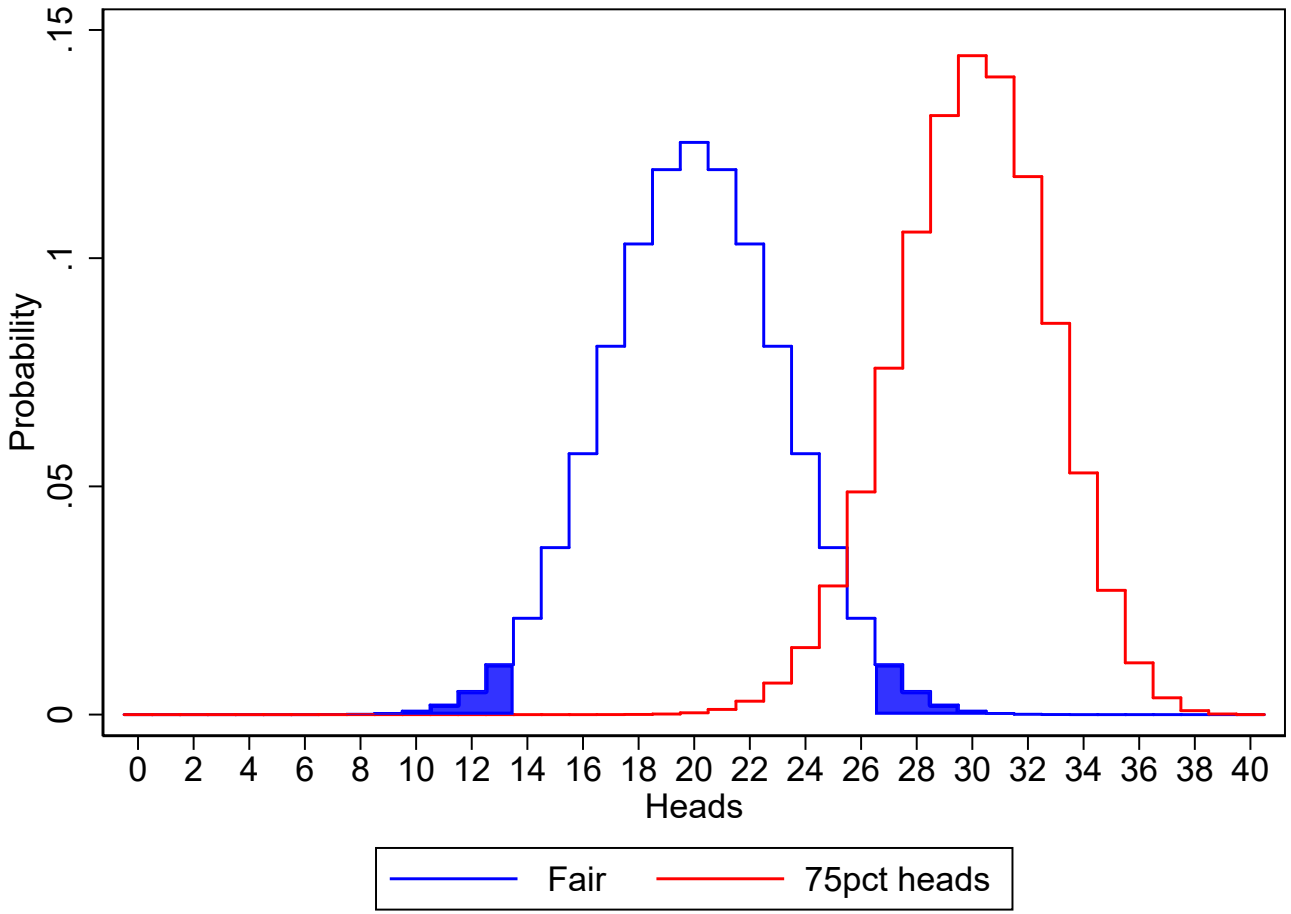

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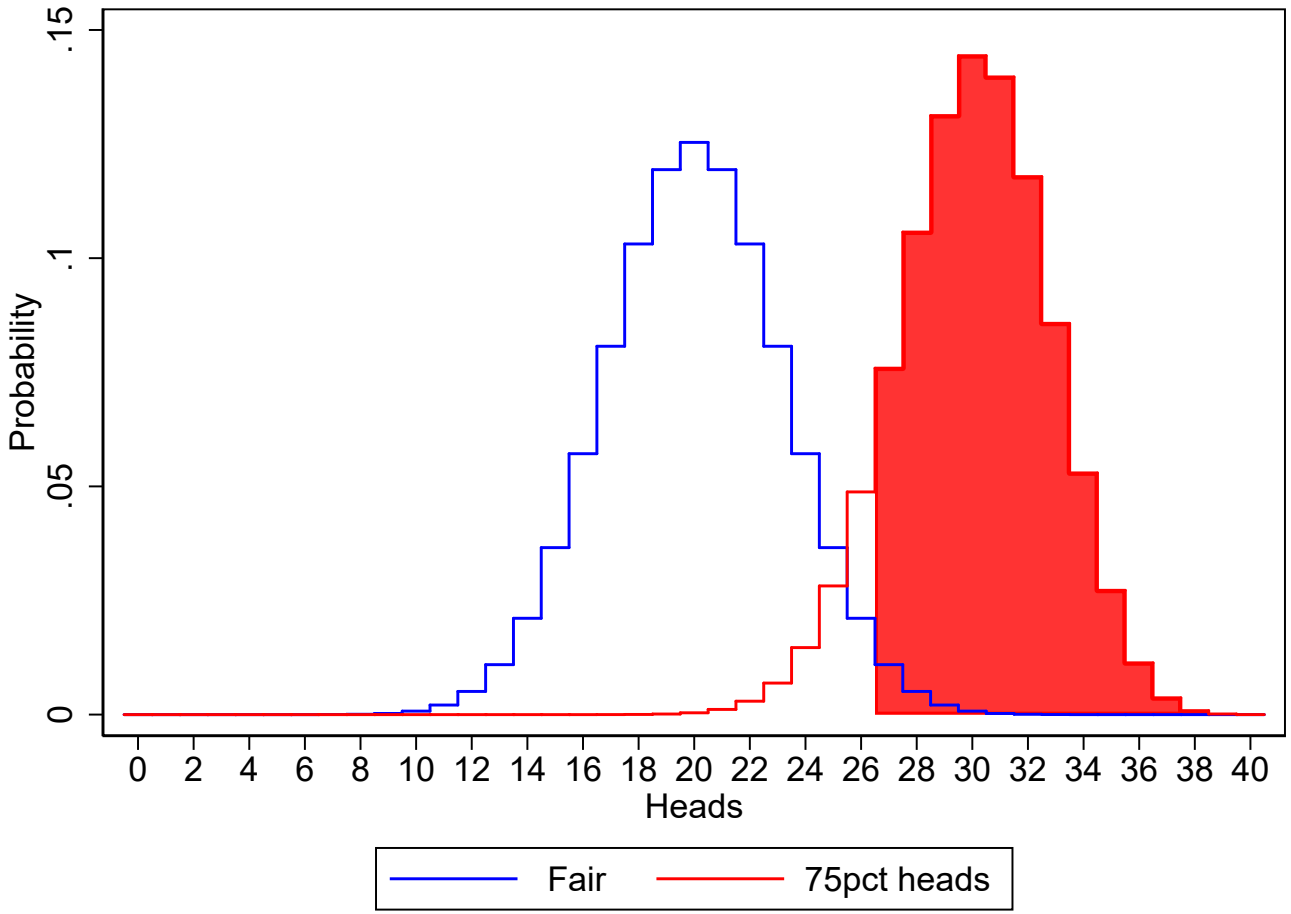
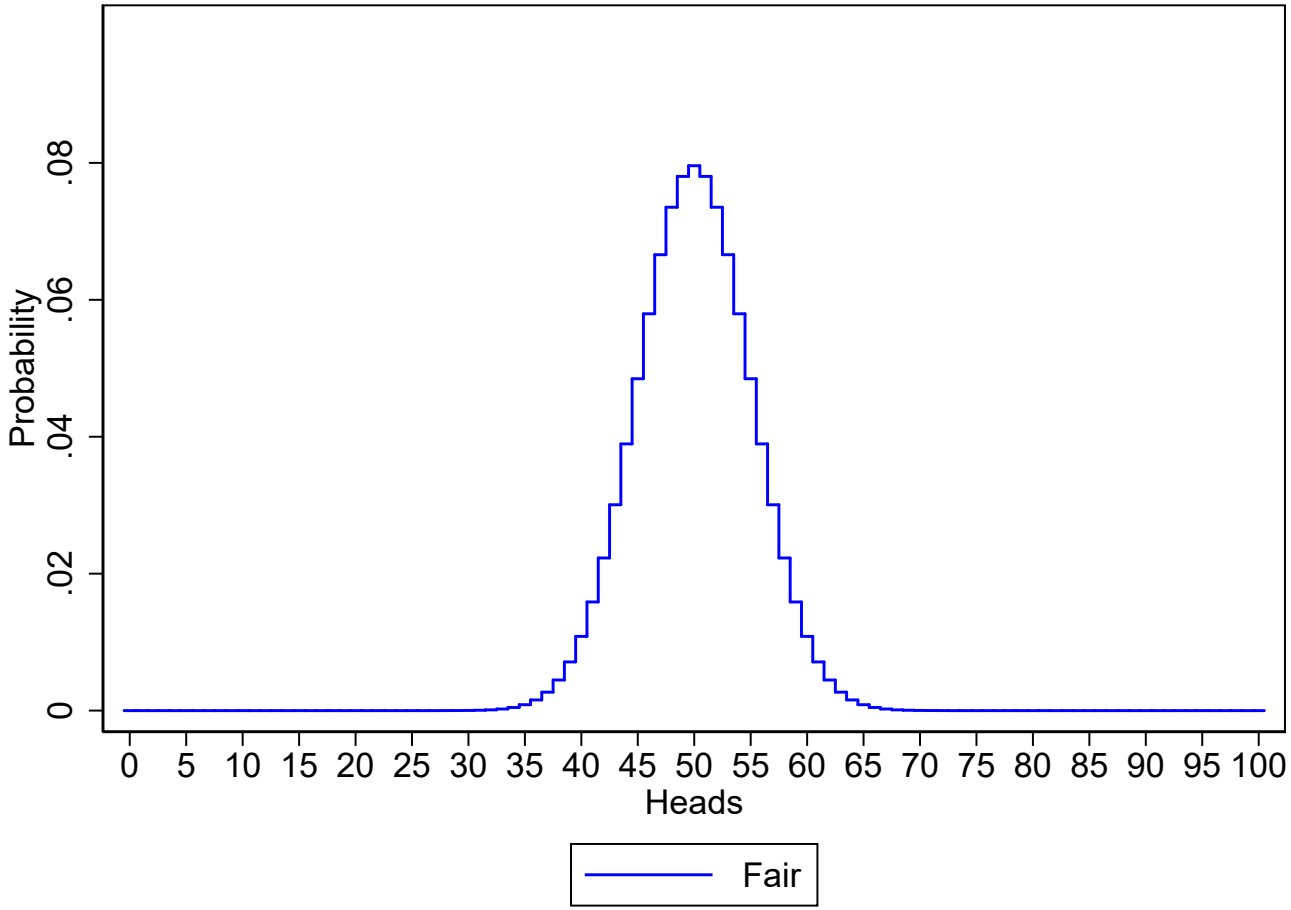


Power: about 0.90

Power with 100 tosses



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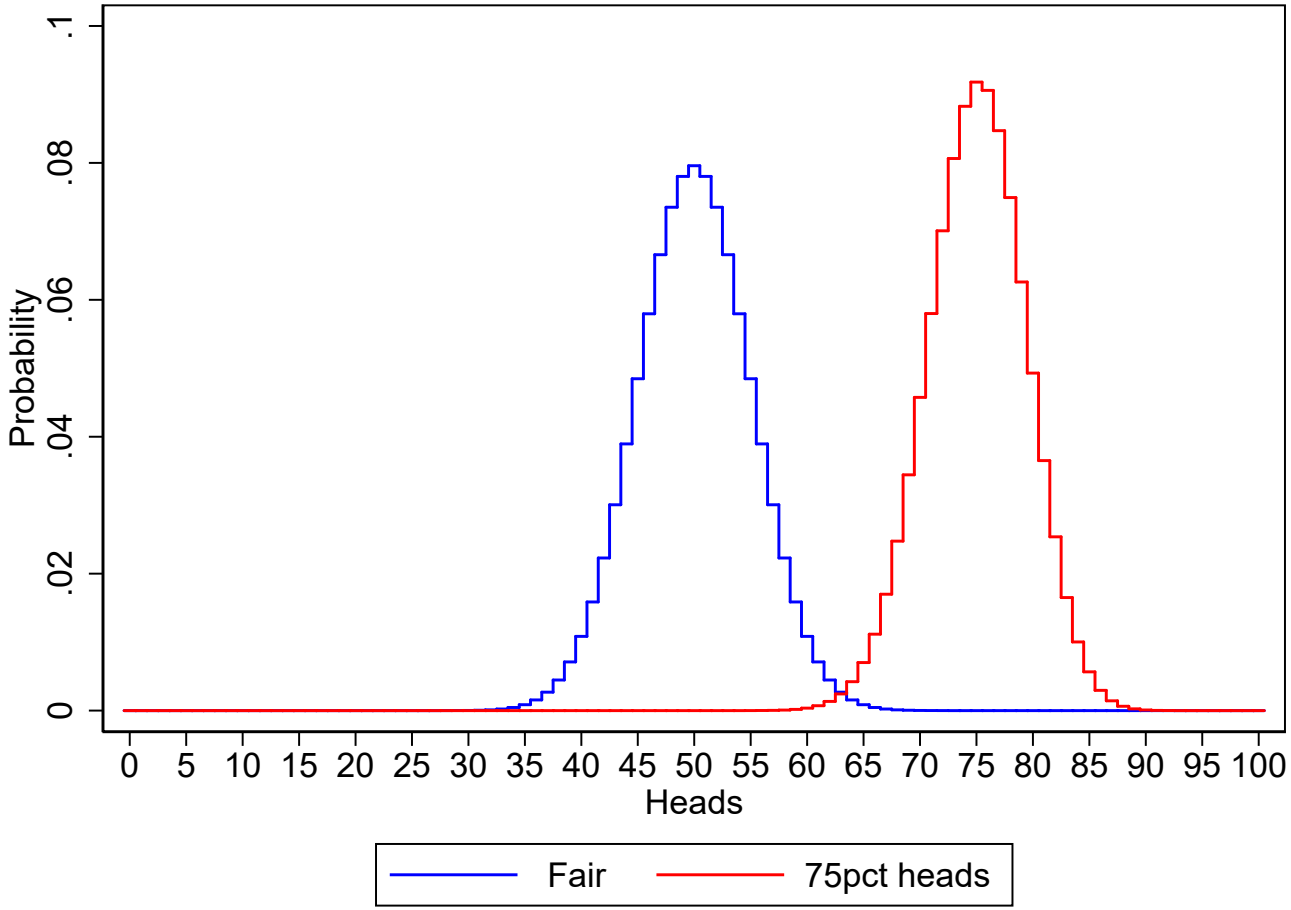

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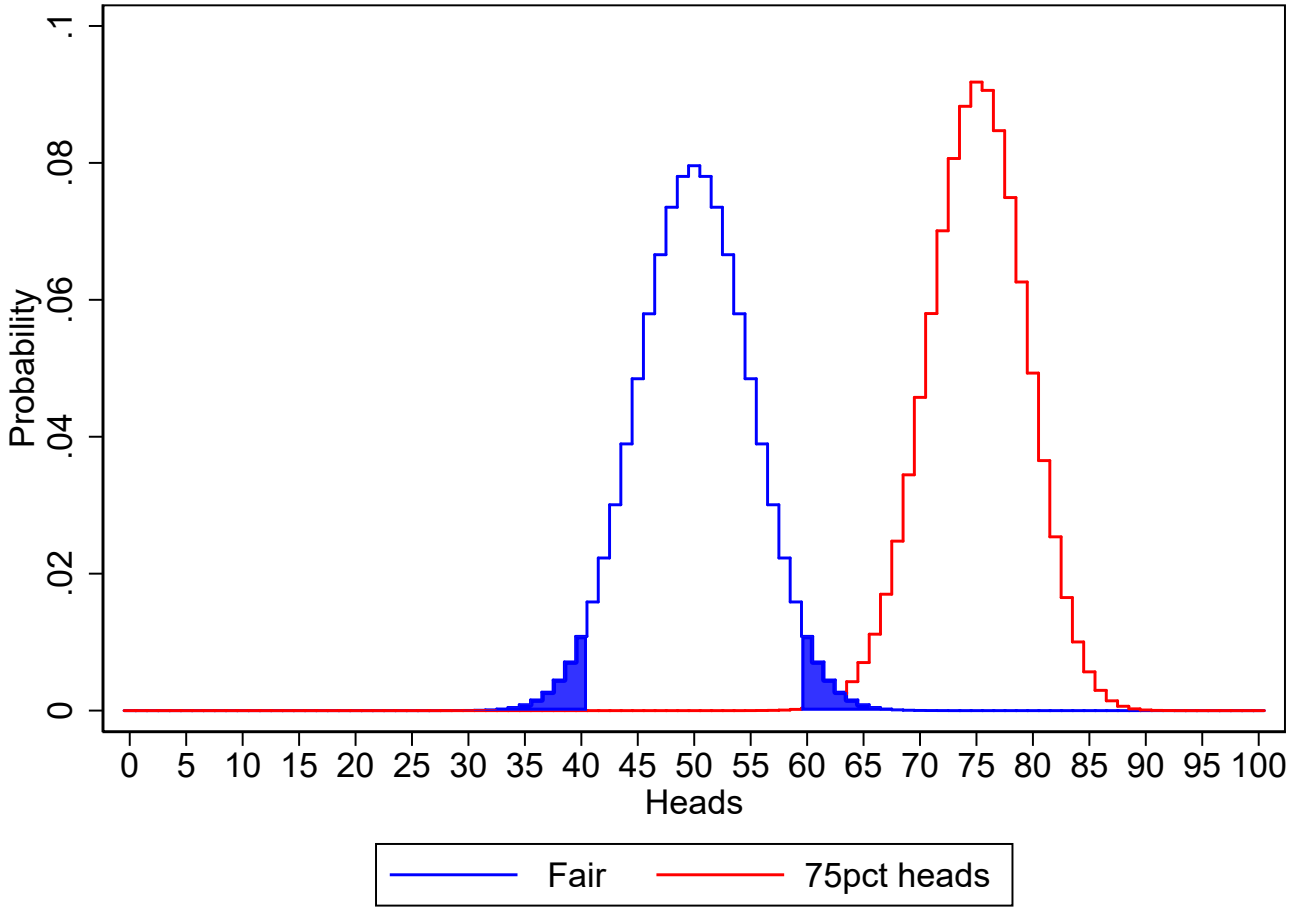

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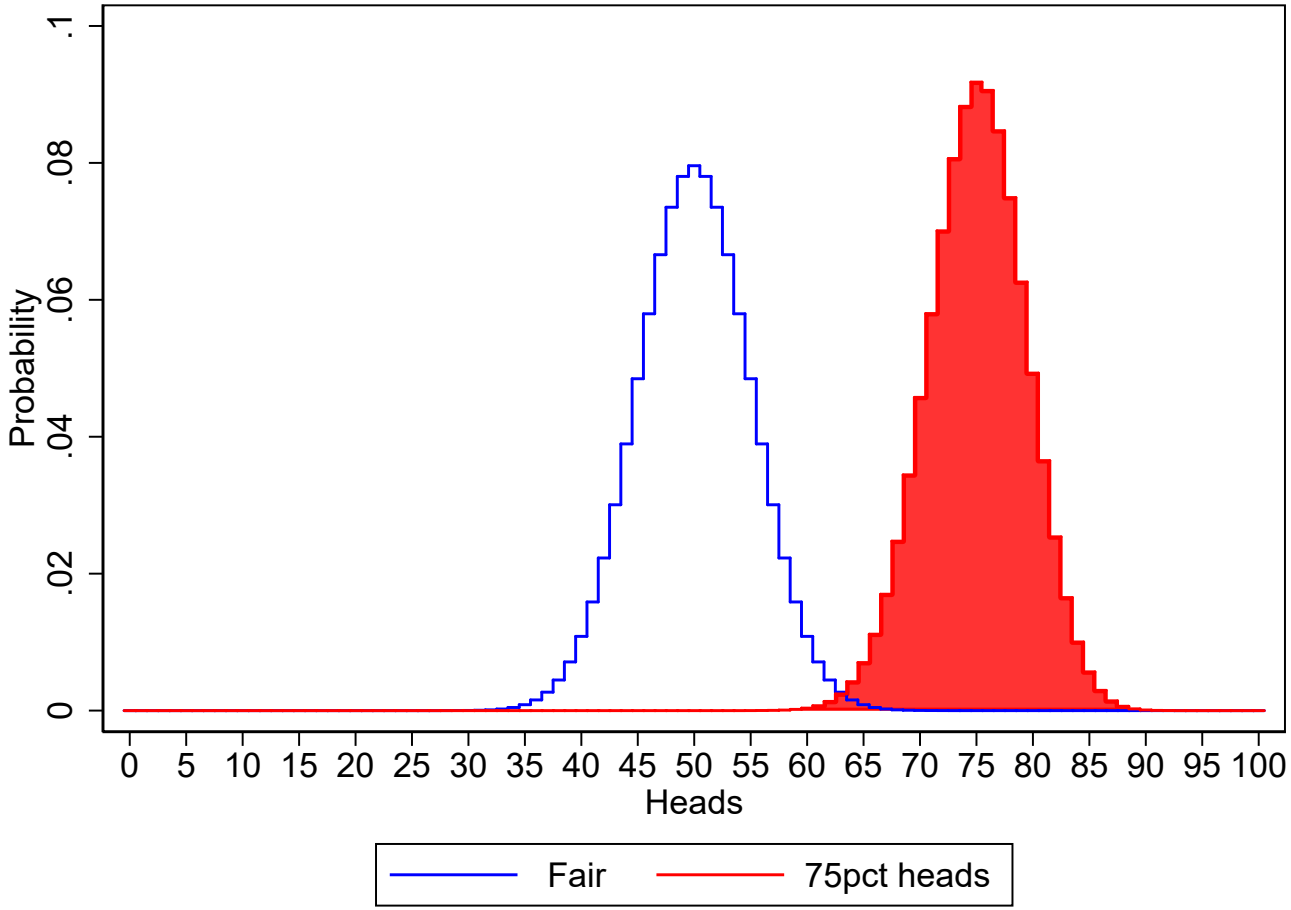
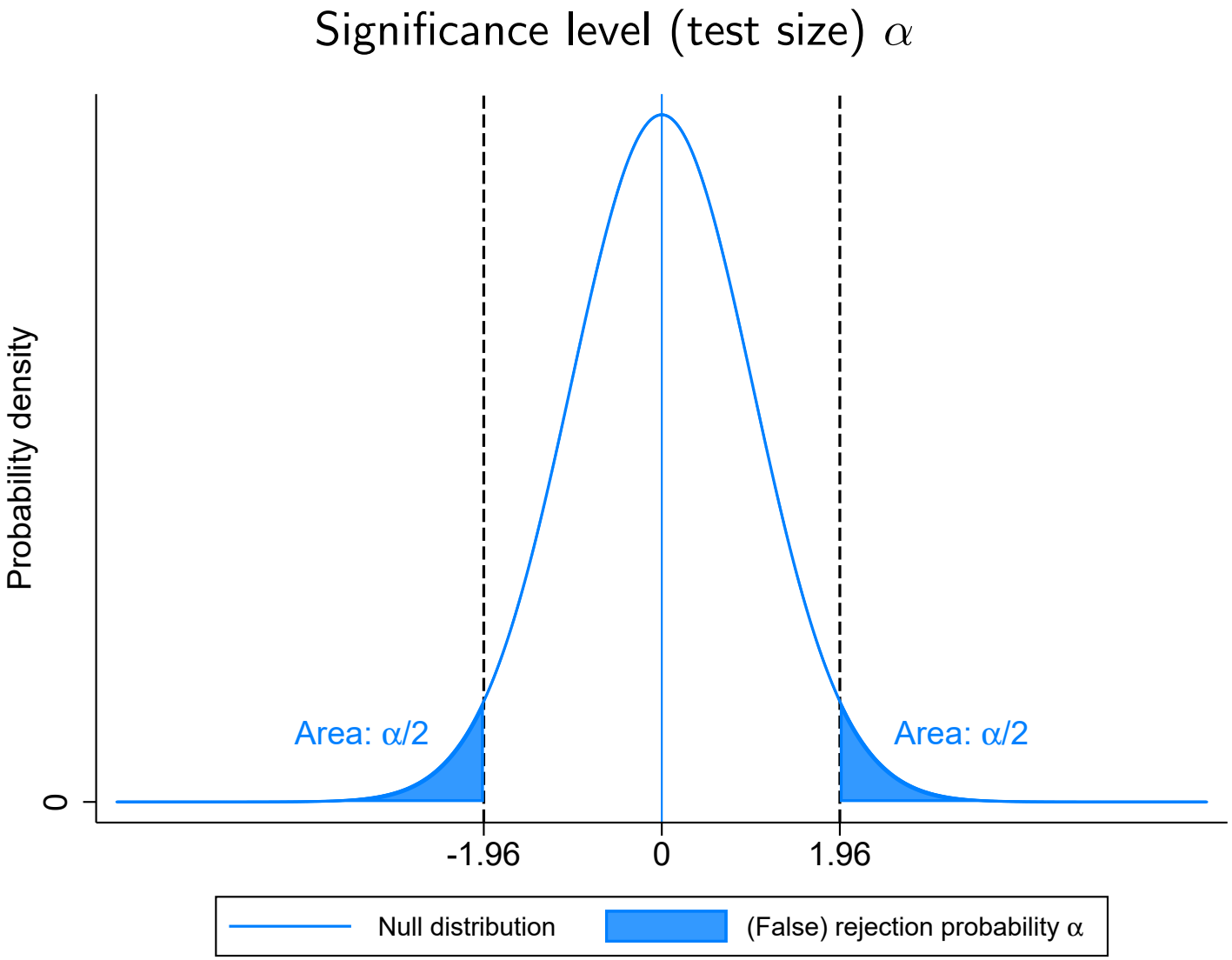


Power: about 0.9997

Rejecting H_0 in critical region



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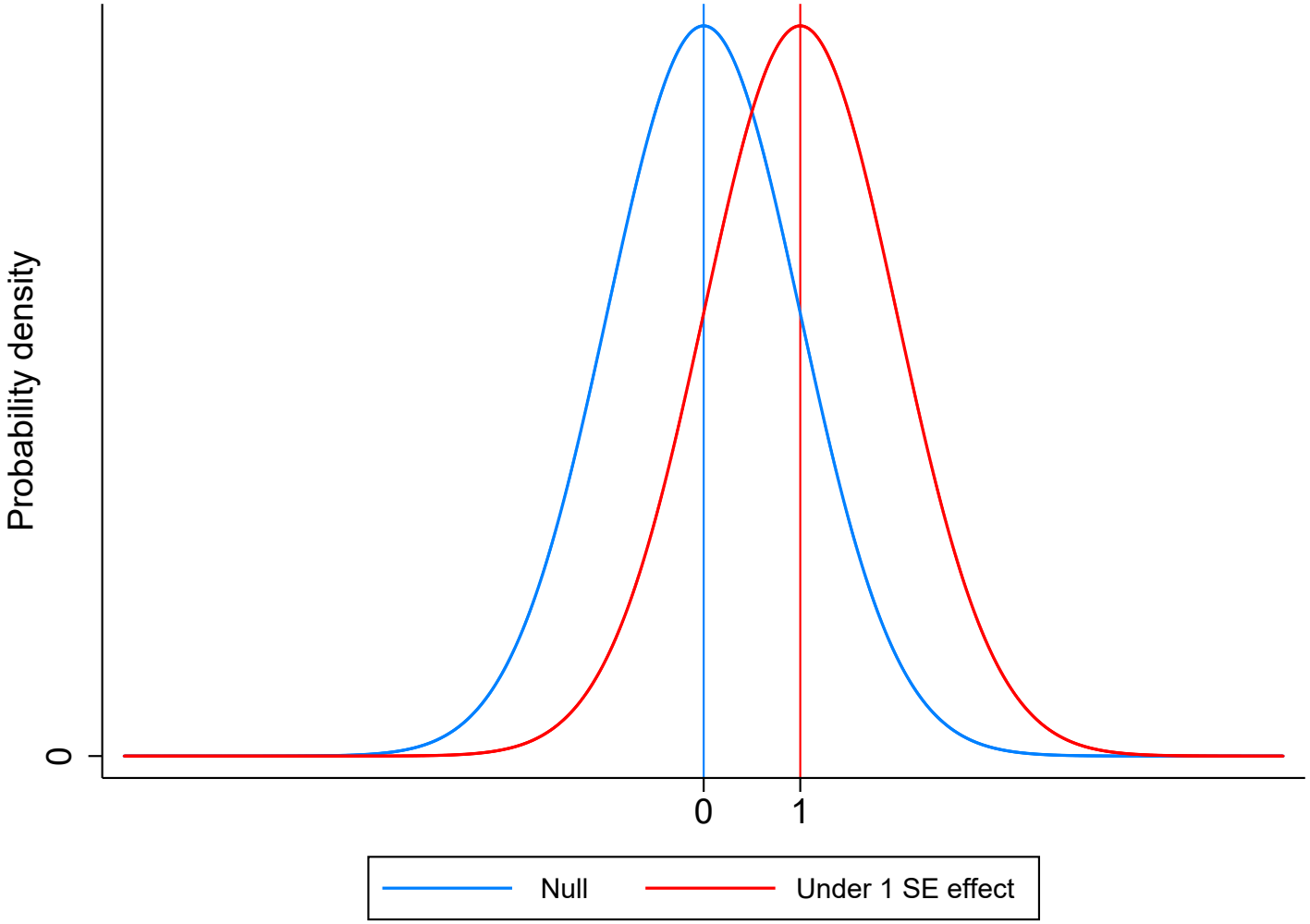

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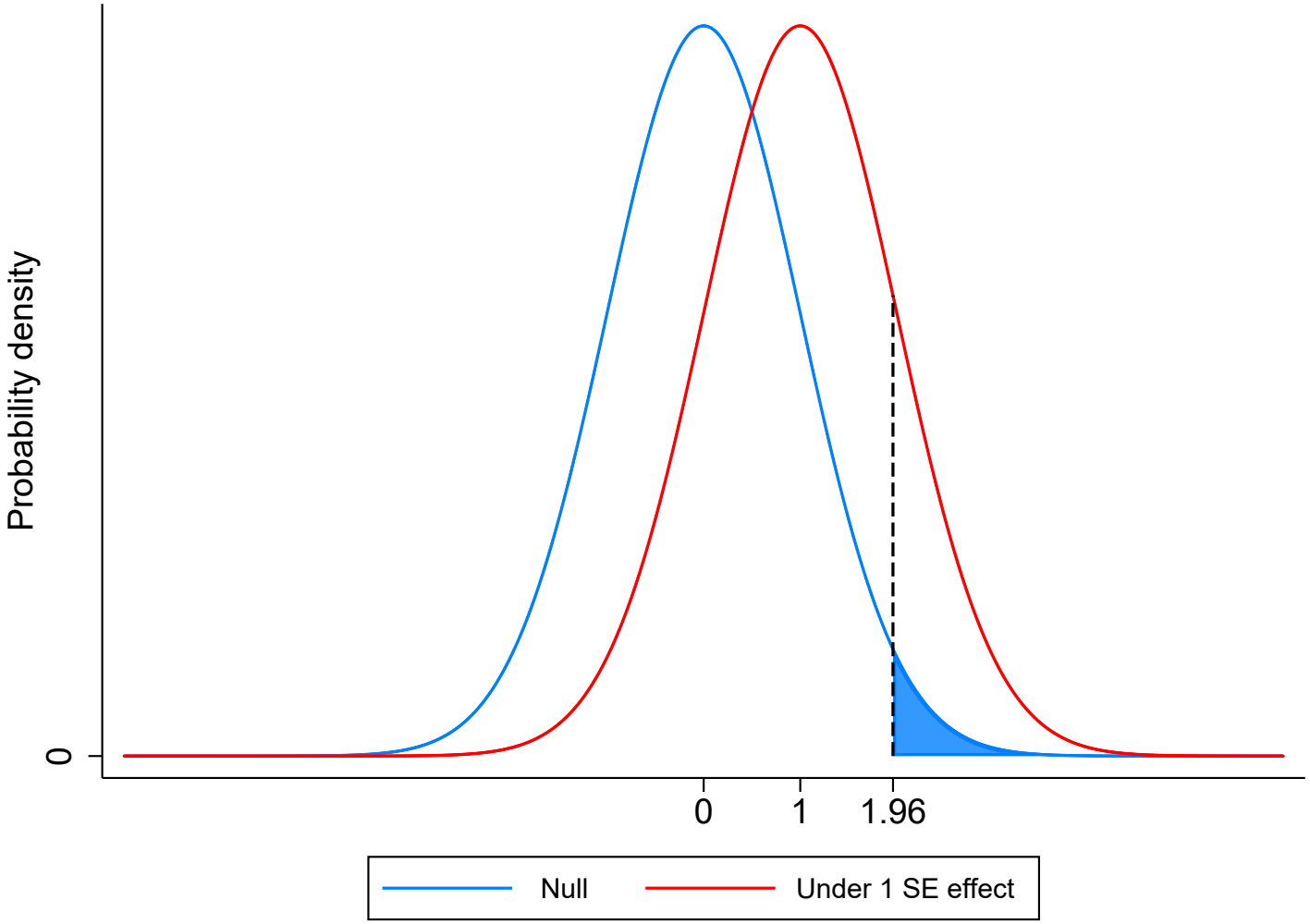

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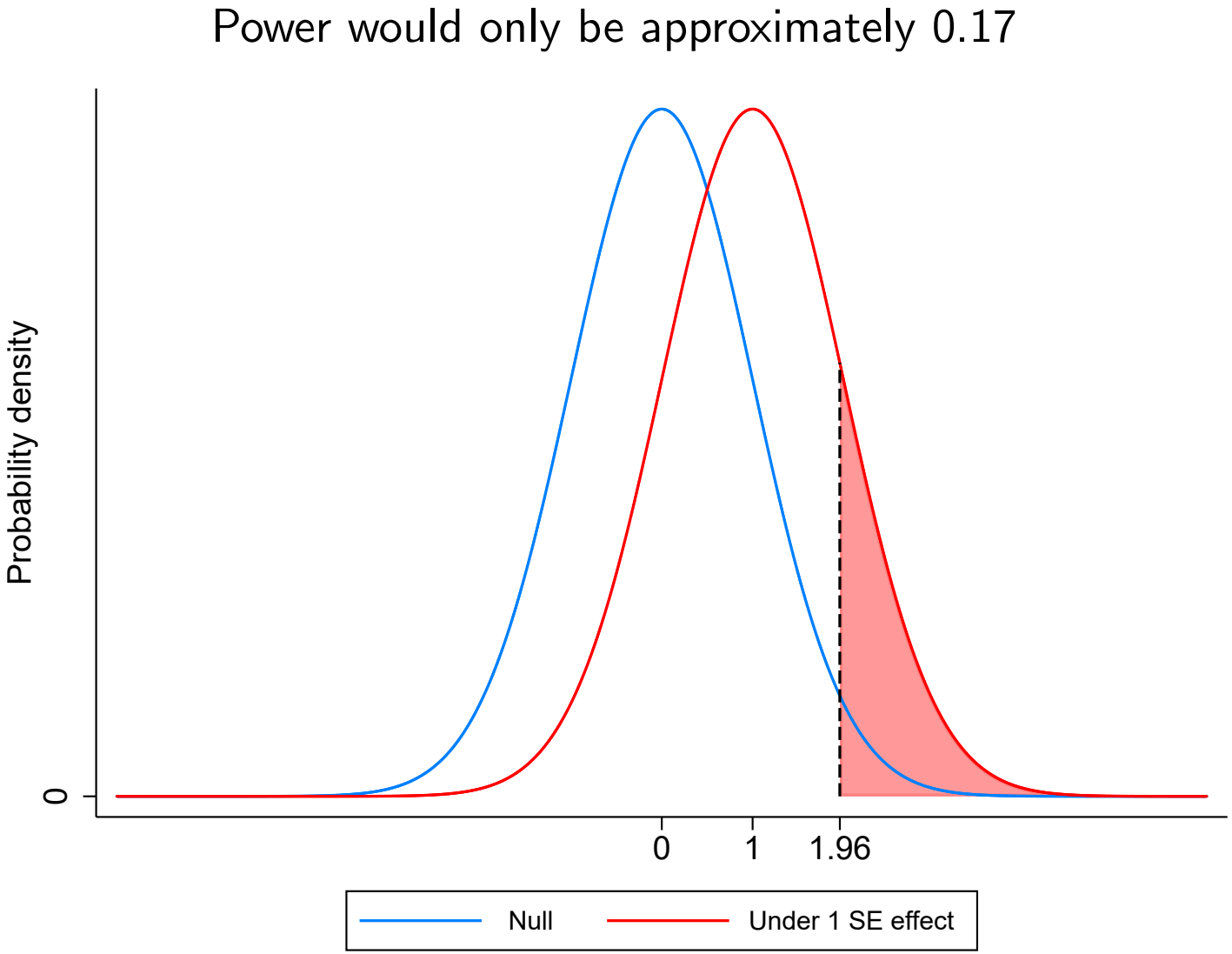

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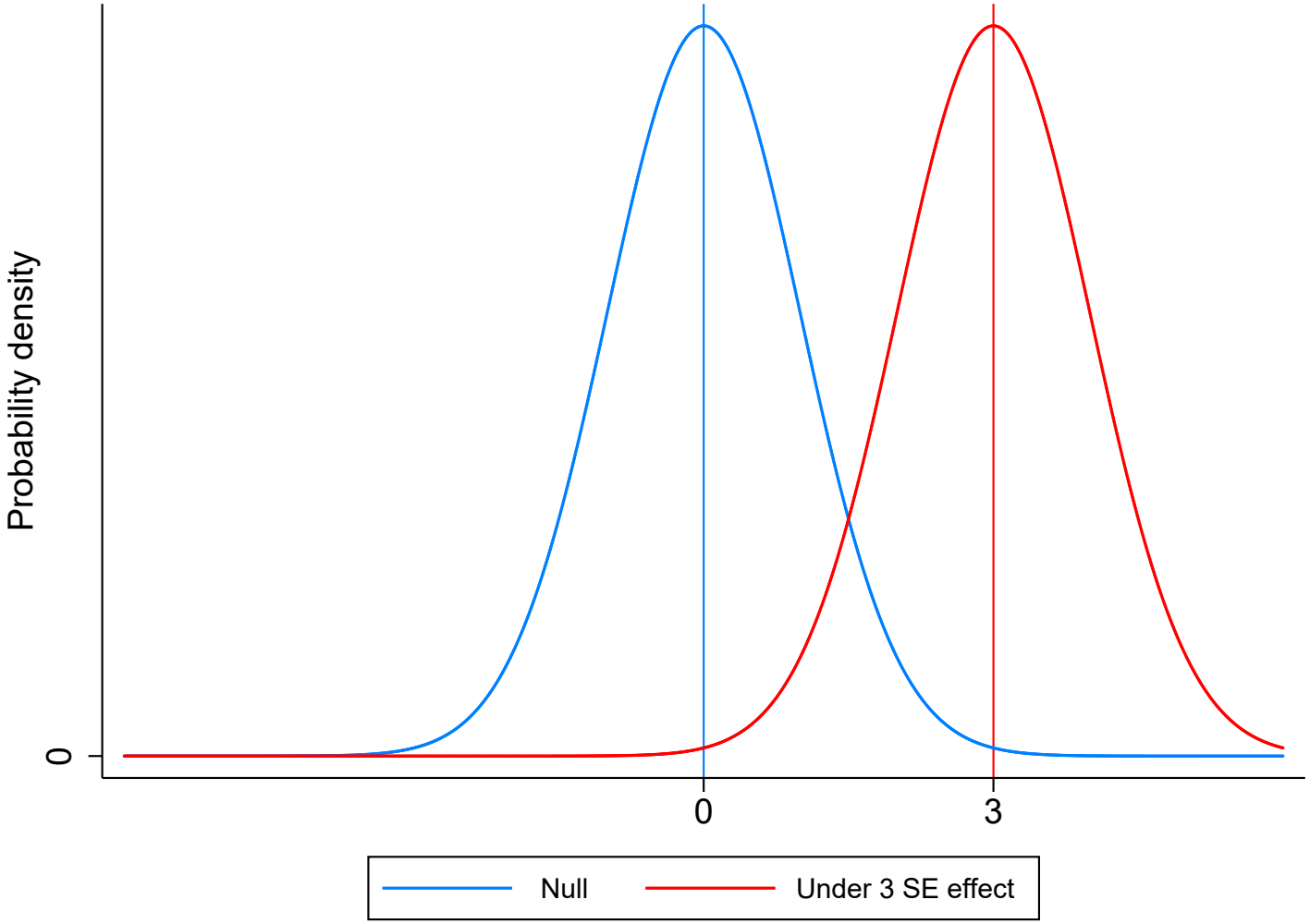

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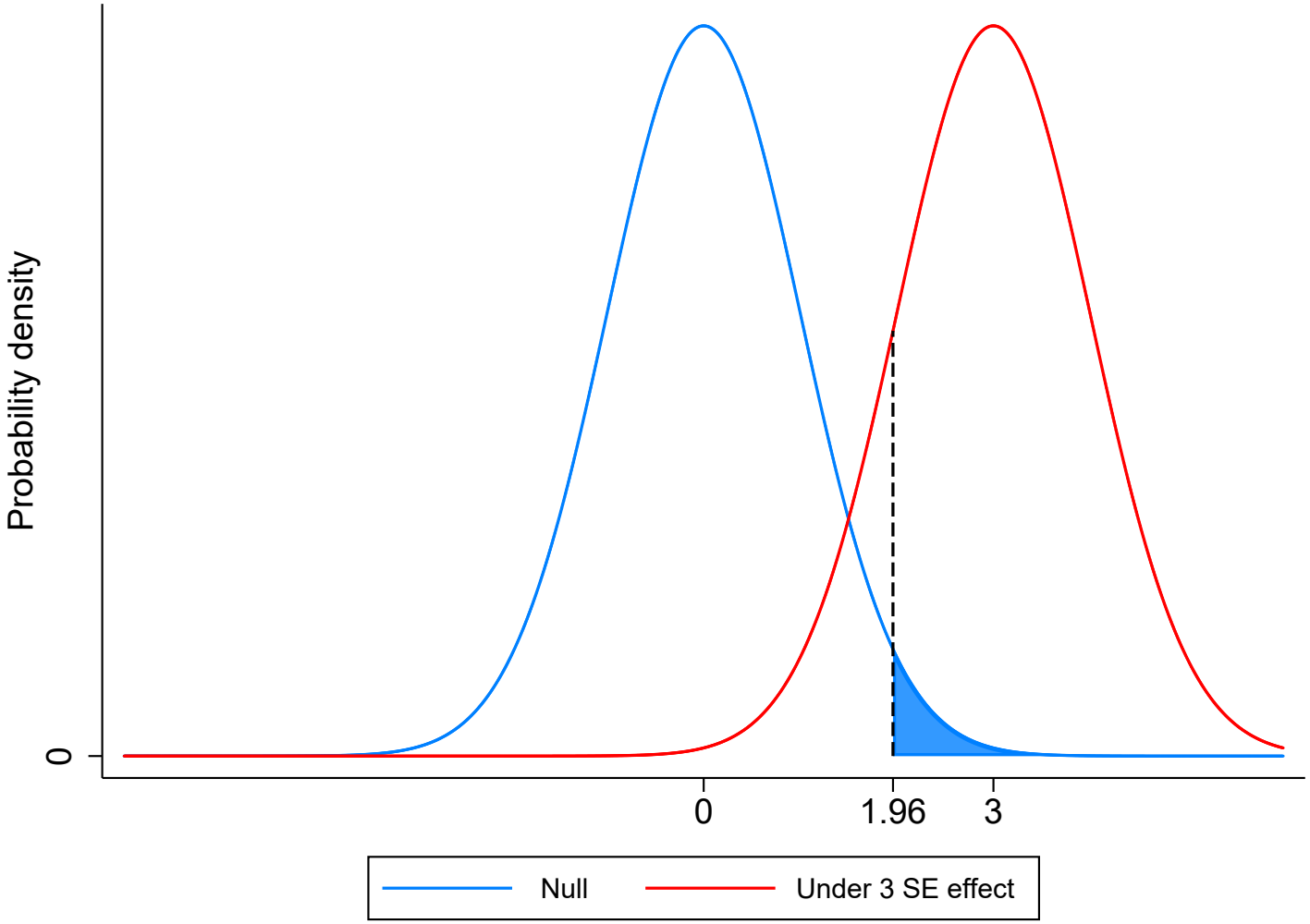

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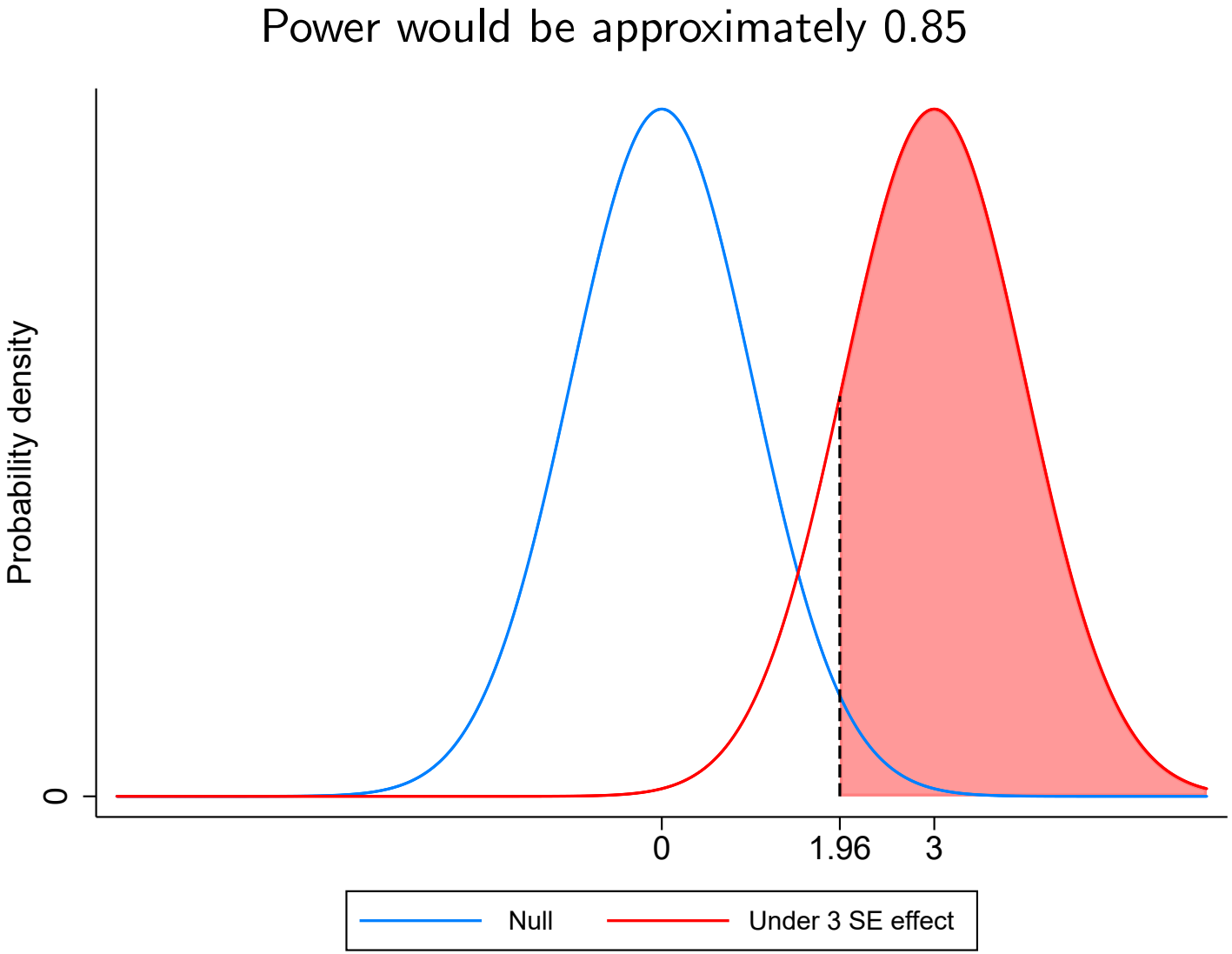
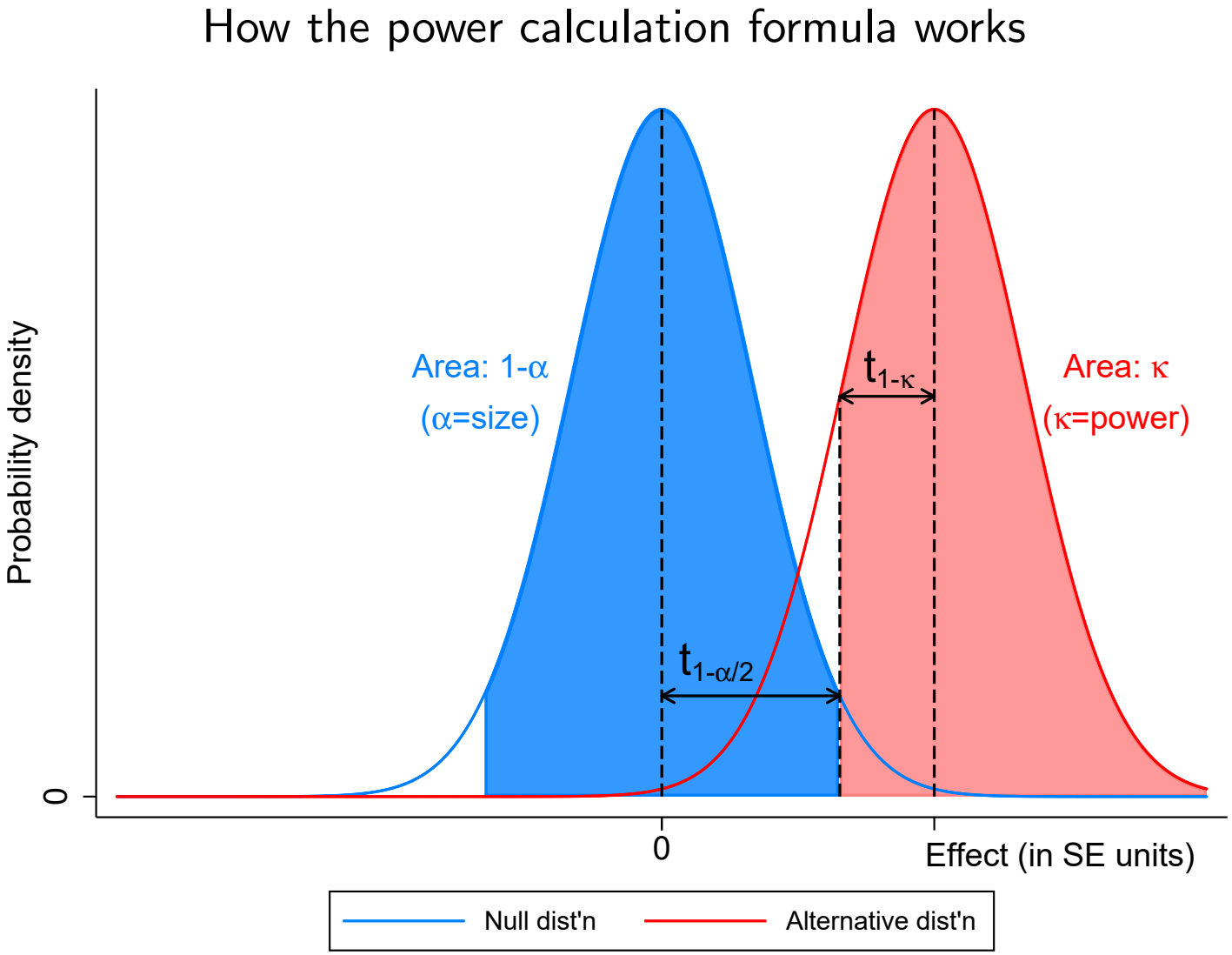


Power calculation, visually



Note: see the related figure in the *Toolkit* paper.

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$$\textit{Effect} > (t_{1-\kappa} + t_{\alpha/2}) SE(\hat{\beta})$$ Notation: $t_{1-p} = p^{th}$ percentile of the t dist'n.

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In practice (Stata): **sampsi**

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$Effect > (t_{1-\kappa} + t_{\alpha/2}) SE(\hat{\beta})$ Notation: $t_{1-p} = p^{th}$ percentile of the t dist'n.

Note that the formula above works no matter the design.
Usually: $\alpha = 0.05$, $\kappa = 0.80$, N is large, so:

$$\text{Minimum Detectable Effect} \approx (0.84 + 1.96) SE(\hat{\beta}) \approx 2.8 SE(\hat{\beta})$$

We focus on sample size. But how would imperfect compliance or baseline data affect this? Below, I continue for the standard RCT case.

$$MDE = (t_{1-\kappa} + t_{\alpha/2}) \sqrt{\frac{1}{P(1-P)}} \sqrt{\frac{\sigma^2}{N}} \quad \approx \quad (z_{1-\kappa} + z_{\alpha/2}) \sqrt{\frac{1}{P(1-P)}} \sqrt{\frac{\sigma^2}{N}}$$

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- Draw on existing data: What is available that could inform your project?

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What does this do?
It depends on how much variation is explained by the group each individual is in.

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This is the formula from before, with $P = 1/2$:

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$$\rho_\epsilon = \frac{\sigma_\nu^2}{\sigma_\nu^2 + \sigma_\eta^2} = \frac{\sigma_\nu^2}{\sigma_\epsilon^2}$$

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Let's think through two pieces of ϵ_i and the variance of their sums; we will need this in just a moment. Within a single study arm (so consider $N/2$ observations). First, the **simple** case, η_{ig}:

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$$\hat{\beta} = \beta + \frac{1}{N/2}\sum_{T}\nu_g + \frac{1}{N/2}\sum_{T}\eta_{ig} - \frac{1}{N/2}\sum_{C}\nu_g - \frac{1}{N/2}\sum_{C}\eta_{ig}$$

$$\begin{aligned}
Var(\hat{\beta}) =& \frac{n_g}{N/2}\sigma_\nu^2 + \frac{1}{N/2}\sigma_\eta^2 + \frac{n_g}{N/2}\sigma_\nu^2 + \frac{1}{N/2}\sigma_\eta^2 \\
=& \frac{4n_g}{N}\sigma_\nu^2 + \frac{4}{N}\sigma_\eta^2 \\
=& \frac{4}{N}\left(n_g\sigma_\nu^2 + \sigma_\eta^2\right) \\
=& \frac{4}{N}\left(n_g\rho_\epsilon\sigma_\epsilon^2 + (1-\rho_\epsilon)\sigma_\epsilon^2\right) = \frac{4}{N}\sigma_\epsilon^2\left((n_g-1)\rho_\epsilon + 1\right)
\end{aligned}$$

$$SE(\hat{\beta}) = \sqrt{4}\sqrt{\frac{\sigma_\epsilon^2}{N}}\sqrt{(n_g-1)\rho_\epsilon + 1} =$$

What happens to the variance of the estimator?

As before,

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Var(\hat{\beta}) =& \frac{n_g}{N/2}\sigma_\nu^2 + \frac{1}{N/2}\sigma_\eta^2 + \frac{n_g}{N/2}\sigma_\nu^2 + \frac{1}{N/2}\sigma_\eta^2 \\
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\end{aligned}$$

$$SE(\hat{\beta}) = \sqrt{4}\sqrt{\frac{\sigma_\epsilon^2}{N}}\sqrt{(n_g-1)\rho_\epsilon + 1} = \sqrt{\frac{1}{P(1-P)}}\sqrt{\frac{\sigma^2}{N}}\sqrt{(n_g-1)\rho_\epsilon + 1}$$

The formula

Scale the effective standard error by:

$$\textit{Design Effect (“Moulton factor”)} = \sqrt{1 + (n_{groupsize} - 1)\rho}$$

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Estimation example: clustered standard errors

Stata:

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$$V_{cluster} = (X'X)^{-1} \sum_{j=1}^{n_c} u_j' u_j (X'X)^{-1}$$

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$$u_j = \sum_{j_{cluster}} e_i x_i$$

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$$\hat{\Omega}_{cl} = (X'X)^{-1} \left(\sum_g X_g' \hat{\Psi}_g X_g \right) (X'X)^{-1}$$

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$$\hat{\Psi}_g = a \hat{e}_g \hat{e}_g' = a \begin{bmatrix} \hat{e}_{1g}^2 & \hat{e}_{1g}\hat{e}_{2g} & \dots & \hat{e}_{1g}\hat{e}_{n_g g} \\ \hat{e}_{2g}\hat{e}_{1g} & \hat{e}_{2g}^2 & \dots & \hat{e}_{2g}\hat{e}_{n_g g} \\ \dots & \dots & \dots & \dots \\ \hat{e}_{n_g g}\hat{e}_{1g} & \hat{e}_{n_g g}\hat{e}_{2g} & \dots & \hat{e}_{n_g g}^2 \end{bmatrix}$$

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Count the terms. diagonal: n_g;

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Estimation example: clustered standard errors

So:

$$E\left[\left(\sum_g X_g' \hat{\Psi}_g X_g\right)\right] =$$

Estimation example: clustered standard errors

So:

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This is a familiar matrix - it is $X'X$!

Estimation example: clustered standard errors

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and thus

$$E\left[\hat{\Omega}_{cl}\right] = E\left[(X'X)^{-1}\left(\sum_g X_g'\hat{\Psi}_g X_g\right)(X'X)^{-1}\right]$$

$$= (1+(n_g-1)\rho)\ (X'X)^{-1}\ \sigma_\epsilon^2$$

Intra-cluster correlation ρ (greek letter "rho")

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But where does this ρ number come from before you have endline data?
Two basic options:

- Consider what might be reasonable assumptions
- Draw on existing data (again):
  What is available that could inform your project?

Intra-class correlations we have known

Data source	**ICC (ρ)**
Madagascar Math + Language	0.5
Busia, Kenya Math + Language	0.22
Udaipur, India Math + Language	0.23
Mumbai, India Math + Language	0.29
Vadodara, India Math + Language	0.28
Busia, Kenya Math	0.62
Busia, Kenya Language	0.43
Busia, Kenya Science	0.35

Duflo, Glennerster, and Kremer (2006) Using Randomization in Development Economics Research: A Toolkit

Data source	**ICC (ρ)**
US Elementary Math, unconditional	0.22
US Elementary Math, rural only, unconditional	0.15
US Elementary Math, rural only, conditional on previous scores	0.12

Hedges & Hedberg (2007), Intraclass correlations for planning group randomized experiments in rural education.

More variations

For discussion or further reading:

- Imperfect compliance with treatment;

May be discussed in later lectures:

Next:

More variations

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- Multiple treatments, multiple testing, attrition

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For discussion or further reading:

- Imperfect compliance with treatment;
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- *"A first comment is that, despite all the precision of these formulas, power calculations involve substantial guess work in practice."*

May be discussed in later lectures:

- Multiple treatments, multiple testing, attrition

Next:

- Actual mechanics of randomization; covariates; stratification

Lecture 7, Part 2:

Design and Balance in Randomized Trials

Besides statistics, registration

Economics: since 2012

Besides statistics, registration

Other disciplines: this example since 2000

BioMed Central
The Open Access Publisher

ISRCTN registry

What is the ISRCTN registry?

ISRCTN is a registry and curated database containing the basic set of data items deemed essential to describe a study at inception, as per the requirements set out by the World Health Organization (WHO) International Clinical Trials Registry Platform (ICTRP) and the International Committee of Medical Journal Editors (ICMJE) guidelines. All study records in the database are freely accessible and searchable and have been assigned an ISRCTN ID.

The registry was launched in 2000, in response to the growing body of opinion in favour of prospective registration of randomised controlled trials (RCTs). Originally ISRCTN stood for 'International Standard Randomised Controlled Trial Number'; however, over the years the scope of the registry has widened beyond randomized controlled trials to include any study designed to assess the efficacy of health interventions in a human population. This includes both observational and interventional trials.

Other registries include non-RCTs, focus on specific fields, etc.

Besides statistics, documentation: CONSORT

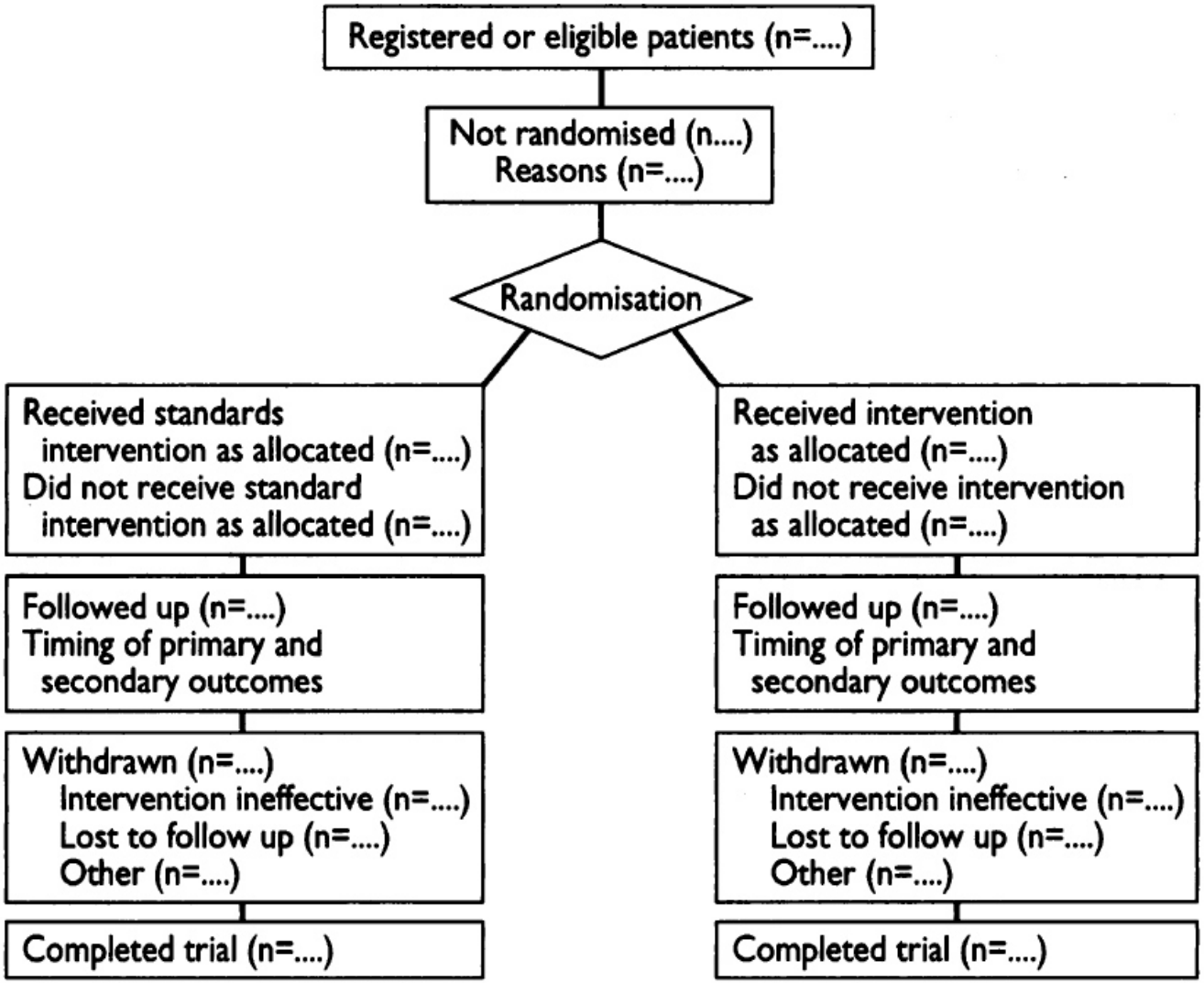
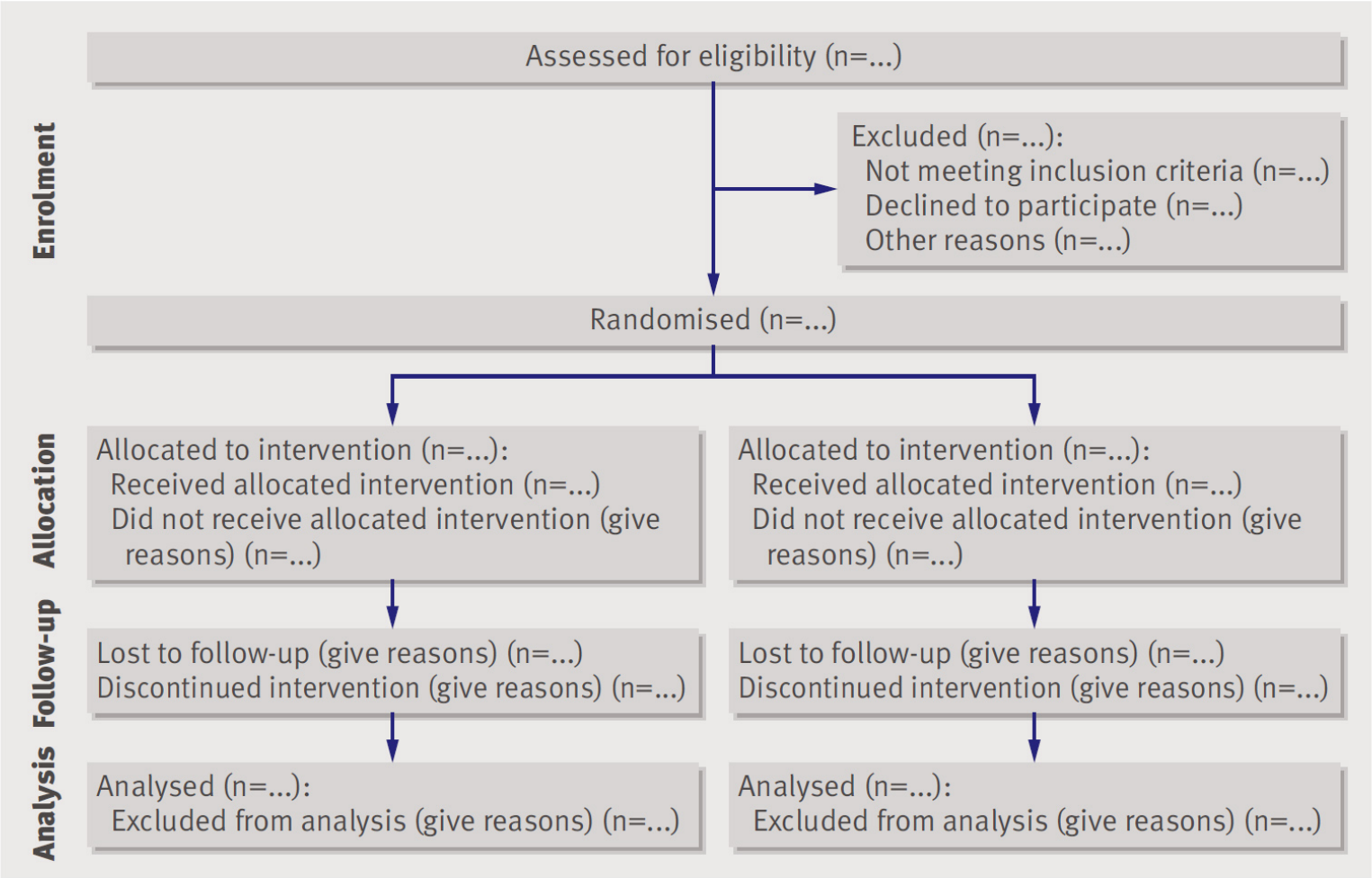


Fig 1—*Flow chart describing progress of patients through randomised trial (reproduced from JAMA)*[9]

Besides statistics, documentation: CONSORT



Flow diagram of the progress through the phases of a parallel randomised trial of two groups (that is, enrolment, intervention allocation, follow-up, and data analysis)

CONSORT-style example from QJE

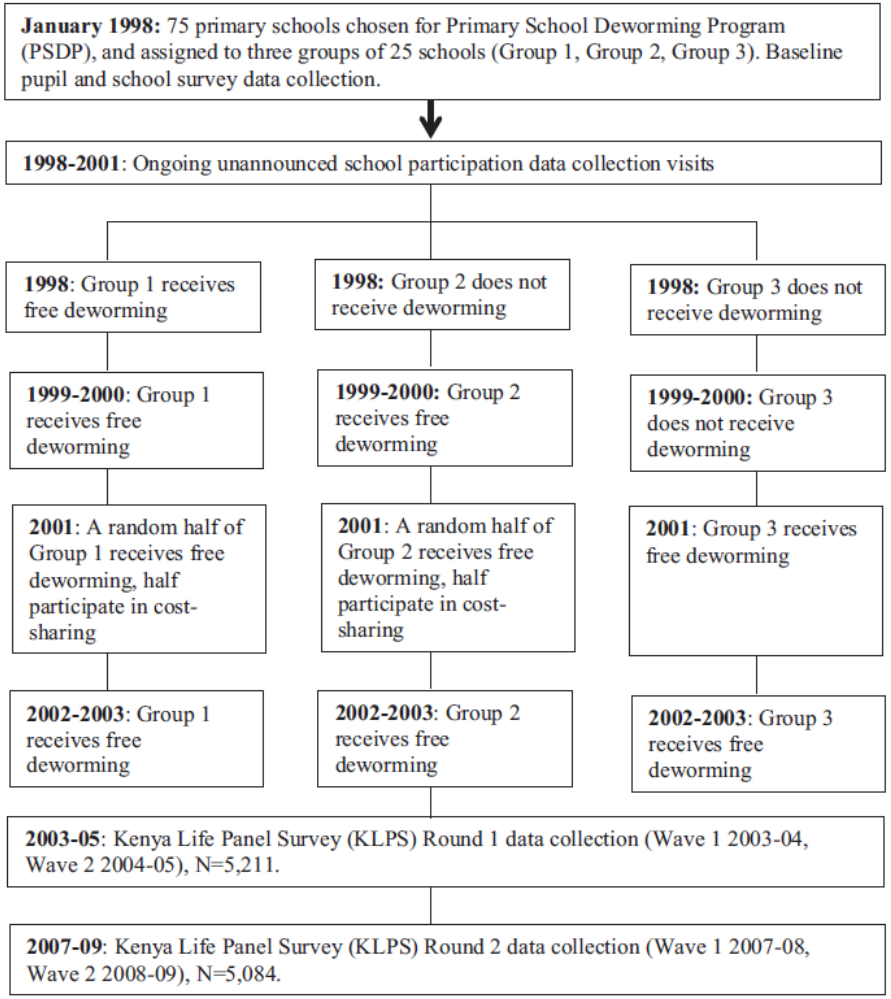


FIGURE I

Project Timeline of the Primary School Deworming Program (PSDP) and the Kenya Life Panel Survey (KLPS)

CONSORT-style example from ECRQ

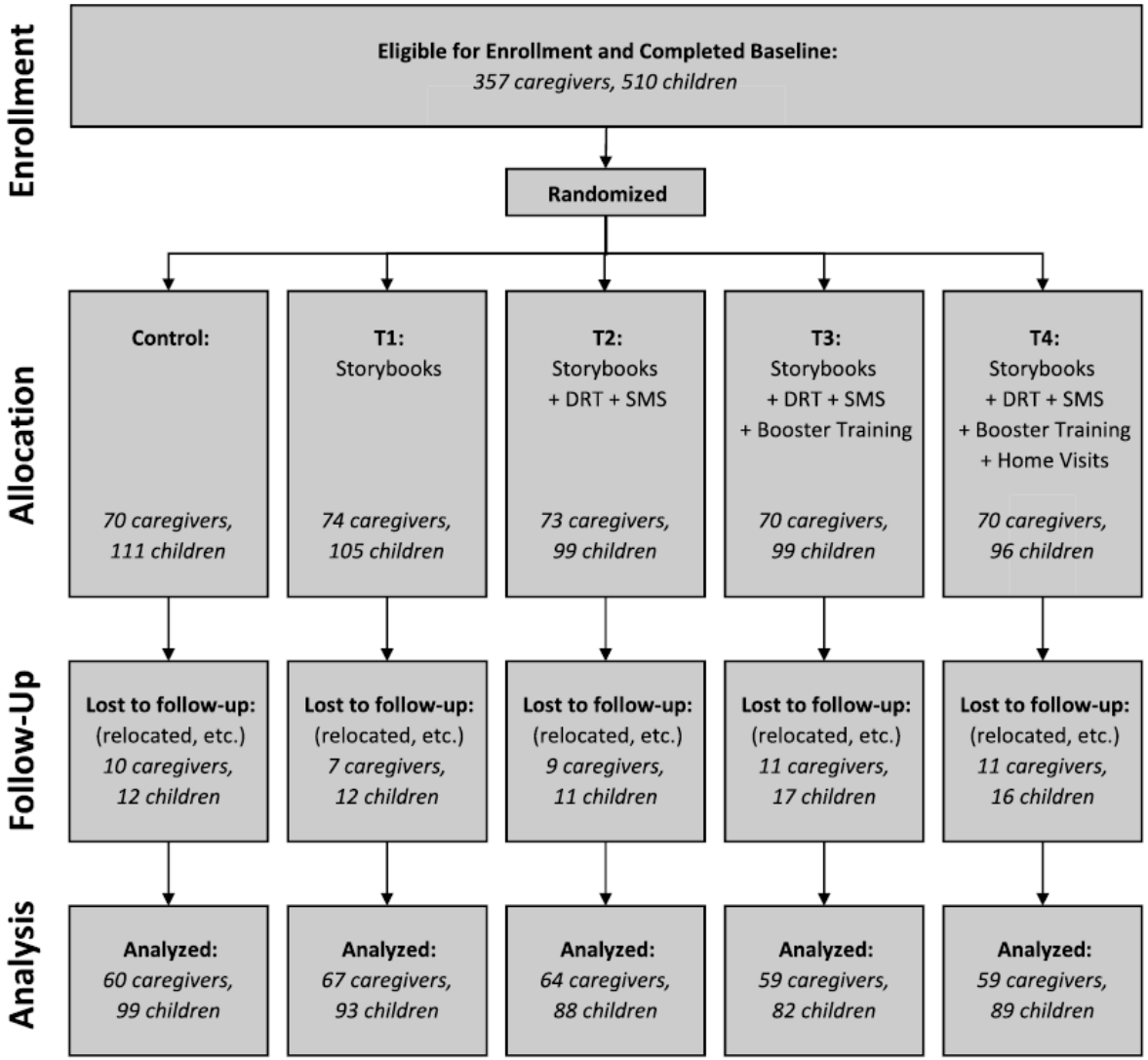


Fig. 1. CONSORT flow diagram.

Besides statistics, documentation: CONSORT

Table 1—*Items that should be included in reports of randomised trials (reproduced from* JAMA*)*[9]

Heading	Subheading	Descriptor
Title		Identify the study as a randomised trial
Abstract		Use a structured format
Introduction		State prospectively defined hypothesis, clinical objectives, and planned subgroup or covariate analyses
Methods	Protocol	Describe Planned study population, together with inclusion or exclusion criteria Planned interventions and their timing Primary and secondary outcome measure(s) and the minimum important difference(s), and indicate how the target sample size was projected Rationale and methods for statistical analyses, detailing main comparative analyses and whether they were completed on an intention to treat basis Prospectively defined stopping rules (if warranted)
	Assignment	Describe Unit of randomisation (for example, individual, cluster, geographic) Method used to generate the allocation schedule Method of allocation concealment and timing of assignment Method to separate the generator from the executor of assignment
	Masking (blinding)	Describe Mechanism (for example, capsules, tablets) Similarity of treatment characteristics (for example, appearance, taste) Allocation schedule control (location of code during trial and when broken) Evidence for successful blinding among participants, person doing intervention, outcome assessors, and data analysts
Results	Participant flow and follow up	Provide a trial profile (fig 1) summarising participant flow, numbers and timing of randomisation assignment, interventions, and measurements for each randomised group
	Analysis	State estimated effect of intervention on primary and secondary outcome measures, including a point estimate and measure of precision (confidence interval) State results in absolute numbers when feasible (for example, 10/20, not 50%) Present summary data and appropriate descriptive and interferential statistics in sufficient detail to permit alternative analyses and replication Describe prognostic variables by treatment group and any attempt to adjust for them Describe protocol deviations from the study as planned, together with the reasons
Discussion		State specific interpretation of study findings, including sources of bias and imprecision (internal validity) and discussion of external validity, including appropriate quantitative measures when possible State general interpretation of the data in light of the totality of the available evidence

Besides statistics, documentation: CONSORT

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Some of these are clearly more applicable to economics than others.

Besides statistics, documentation: CONSORT

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Bruhn and McKenzie - Approach

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A survey of practitioners, then **six datasets**:

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- Learning & Educational Achievement Project: height z-score

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> "The set of outcomes we have chosen spans a range of the ability of the baseline variables to predict future outcomes. At one end is microenterprise profits in Sri Lanka, where baseline profits and 6 baseline individual and firm characteristics explain only 12.2 percent of the variation in profits 6 months later. ... The math test scores and height z-scores in the LEAPS data have the most variation explained by baseline characteristics, with 43.6 percent of the variation in follow-up test scores explained by the baseline test score and 6 baseline characteristics."

Bruhn and McKenzie - Recommendation 1

"Better reporting of the **method of random assignment** is needed. This should include a description of:

a. Which randomization method was used and why.

b. Which variables were used for balancing?

c. For stratification, how many strata were used?

d. For rerandomization, which cutoff rules were used?

This is particularly important for experiments with small samples, where the randomization method makes more difference."

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(Obvious in retrospect?)

Bruhn and McKenzie - Recommendation 2

"Clearly describe **how the randomization was carried out** in practice.
a. Who performed the randomization?
b. How was the randomization done (coin toss, random number generator, etc.)?
c. Was the randomization carried out in public or private?"

Bruhn and McKenzie - Recommendation 3

"**Re-think the common use of rerandomization.**
Our simulations find pair-wise matching to generally perform as well, or better, than rerandomization in terms of balance and power, and like rerandomization, matching allows balance to be sought on more variables than possible under stratification. Adjusting for the method of randomization is statistically cleaner with matching or stratification than with rerandomization."

Bruhn and McKenzie - Recommendation 4

"When deciding which variables to balance on, strongly consider the **baseline outcome variable and geographic region dummies, in addition to variables desired for subgroup analysis.**"

Bruhn and McKenzie - Recommendation 5

"**Be aware that over-stratification can lead to a loss of power** in extreme cases. This is because using a large number of strata involves a downside in terms of loss in degrees of freedom when estimating standard errors, possibly more cases of missing observations, and odd numbers within strata when stratification is used."

Bruhn and McKenzie - Recommendation 6

"**As ye randomize, so shall ye analyze.**" (Include dummies for strata in analysis.) "Similarly, pair dummies should be included for matched randomization, or linear variables used for rerandomizations."

Bruhn and McKenzie - Recommendation 7

"In the ex post analysis, **do not automatically control for baseline variables that show a statistically significant difference in means.** The previous literature, and our simulations, suggest that it is a better rule to control for variables that are thought to influence follow-up outcomes, independent of whether their difference in means is statistically significant or not.

Bruhn and McKenzie - Recommendation 7

"In the ex post analysis, **do not automatically control for baseline variables that show a statistically significant difference in means.** The previous literature, and our simulations, suggest that it is a better rule to control for variables that are thought to influence follow-up outcomes, independent of whether their difference in means is statistically significant or not. ... One should still be cautious in the use of ex post controls, given the potential for finite-sample bias if treatment heterogeneity is correlated with the square of these covariates."

McKenzie (2012)

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Activities