ECON 626: Applied Microeconomics

Lecture 2:

Regression Basics and Heteroskedasticity

Professors: Pamela Jakiela and Owen Ozier

One piece of probability/statistics

Variance of mean

Recall: For any **independent** random variables X and Y:

$$Var(X + Y) =$$

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Thus, for independent random variables X_i each with variance σ^2:

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And so

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Linear Algebra (quick review)

Multiplication

$$k \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

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Four (simple) equations, four unknowns.

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Four (simple) equations, four unknowns.

$$\begin{bmatrix} D_{11} & 0 \\ 0 & D_{22} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{D_{11}} & 0 \\ 0 & \frac{1}{D_{22}} \end{bmatrix}$$

Multiplication, continued

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So inverting a diagonal matrix is easy. A general formula?

Inverses (2x2 case)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

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Transpose

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. Let $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. $AB = ?$

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$BA =$ "conformability error"

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Thus, B'A' =(AB)'. (Note: A' is sometimes written A^T.)

Regression

Recall the basic regression (estimation) formula

$$\hat{\beta} = (X'X)^{-1}X'y$$

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But what is $(X'X)^{-1}$? What does $(X'X)^{-1}X'$ do to y?

Matrices' easy interpretation in "treatment" context

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Suppose that half of N observations have $D_i = 1$ and half have $D_i = 0$.

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Matrices' easy interpretation in "treatment" context

Suppose that we are interested in the relationship between outcome Y_i and a treatment indicator D_i. Regress the outcome on... the treatment indicator and a constant.

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$$= \begin{bmatrix} \sum_T Y_i \\ \\ \sum_T Y_i + \sum_C Y_i \end{bmatrix}$$

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Matrices' easy interpretation in "treatment" context

We can now compute:

$$\hat{\beta} = (X'X)^{-1}X'y$$

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We just ran a regression.

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We just ran a regression. Now, on to the standard error!
What will the dimensions of the variance-covariance matrix be?

Homoskedastic error

Recall the basic regression (estimation) formula

What is the variance of $\hat{\beta} = (X'X)^{-1}X'y$?
First, re-write $\hat{\beta}$:

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$$\begin{aligned}\hat{\beta} = (X'X)^{-1}X'y \quad &= (X'X)^{-1}X'(X\beta + e) \\ &= \beta + (X'X)^{-1}X'e\end{aligned}$$

Structure of the error term, homoskedasticity

Ways of writing second term, $(X'X)^{-1}X'e$:

$$(X'X)^{-1}X'u \qquad (CT\ 4.11)\ with\ E[u|X] = 0\ (assumption\ ii\ p.73)$$

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Before proceeding to estimate variance, independent observations (CT p.73 assumption ii) (assumptions and implication):

$$E[uu'|X] = \Omega = Diag[\sigma_i^2]$$

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$$\hat{\Omega} = \hat{\sigma}^2 I$$

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(In this example, $K=2$.)

The variance-covariance matrix

For the variance, we write this quadratic form of the estimation error:

$$\left((X'X)^{-1}X'u\right)\ \left((X'X)^{-1}X'u\right)'$$

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We have an estimator for the basic variance-covariance matrix under homoskedasticity.

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We have an estimator for the basic variance-covariance matrix under homoskedasticity. What about heteroskedasticity?

Heteroskedasticity

Structure of the error term, revisited

Ways of writing second term, $(X'X)^{-1}X'e$:

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So a reasonable estimator:

$$\hat{\Omega} = Diag[\hat{u}_i^2]\ (CT\ notation)\ = Diag[\hat{e}_i^2]\ (AP\ notation)$$

Why heteroskedasticity?

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Note that AP 3.1.7 is written in expectations, in a formulation that leads to the variance of $\sqrt{N} \cdot \hat{\beta}$, just as CT 4.17 does:

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$$(\textit{notation swap}) \quad E[X_i X_i']^{-1} E[X_i X_i' e_i^2] E[X_i X_i']^{-1} \qquad (AP\ 3.1.7)$$

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$$X'\hat{\Omega}X$$

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$$= \begin{bmatrix} 0 & 0 & \dots & 0 & 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 & 1 & 1 & \dots & 1 \end{bmatrix}$$

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$$= \begin{bmatrix} 0 & 0 & ... & 0 & 1 & 1 & ... & 1 \\ 1 & 1 & ... & 1 & 1 & 1 & ... & 1 \end{bmatrix} Diag[\hat{u}_i^2]$$

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$$= \begin{bmatrix} 0 & 0 & ... & 0 & 1 & 1 & ... & 1 \\ 1 & 1 & ... & 1 & 1 & 1 & ... & 1 \end{bmatrix} Diag[\hat{u}_i^2] \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ ... & ... \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ ... & ... \\ 1 & 1 \end{bmatrix}$$

My big fat greek ... diagonal matrix

$$\hat{\Omega}X = \begin{bmatrix} \hat{u}_1^2 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & \hat{u}_2^2 & \dots & 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \hat{u}_{\frac{N}{2}}^2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \hat{u}_{\frac{N}{2}+1}^2 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & \hat{u}_{\frac{N}{2}+2}^2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & \hat{u}_N^2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ \dots & \dots \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ \dots & \dots \\ 1 & 1 \end{bmatrix}$$

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$$\hat{\Omega}X = \begin{bmatrix} 0 & \hat{u}_1^2 \\ 0 & \hat{u}_2^2 \\ \dots & \dots \\ 0 & \hat{u}_{\frac{N}{2}}^2 \\ \hat{u}_{\frac{N}{2}+1}^2 & \hat{u}_{\frac{N}{2}+1}^2 \\ \hat{u}_{\frac{N}{2}+2}^2 & \hat{u}_{\frac{N}{2}+2}^2 \\ \dots & \dots \\ \hat{u}_N^2 & \hat{u}_N^2 \end{bmatrix}$$

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$$X'\hat{\Omega}X = \left[\begin{array}{cccccccc} 0 & 0 & \ldots & 0 & 1 & 1 & \ldots & 1 \\ 1 & 1 & \ldots & 1 & 1 & 1 & \ldots & 1 \end{array}\right] \left[\begin{array}{cc} 0 & \hat{u}_1^2 \\ 0 & \hat{u}_2^2 \\ \ldots & \ldots \\ 0 & \hat{u}_{\frac{N}{2}}^2 \\ \hat{u}_{\frac{N}{2}+1}^2 & \hat{u}_{\frac{N}{2}+1}^2 \\ \hat{u}_{\frac{N}{2}+2}^2 & \hat{u}_{\frac{N}{2}+2}^2 \\ \ldots & \ldots \\ \hat{u}_N^2 & \hat{u}_N^2 \end{array}\right]$$

$$=$$

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$$= \begin{bmatrix} \quad \end{bmatrix}$$

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$$= \begin{bmatrix} \sum_{i=\frac{N}{2}+1}^{N} \hat{u}_i^2 & \\ & \\ & \end{bmatrix}$$

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$$(X'X)^{-1}X'\hat{\Omega}X(X'X)^{-1} \quad (CT\ 4.21)$$

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$$\left(\frac{2}{N}\right)\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \sum\limits_T \hat{u}^2 & \sum\limits_T \hat{u}^2 \\ \sum\limits_T \hat{u}^2 & \sum\limits_T \hat{u}^2 + \sum\limits_C \hat{u}^2 \end{bmatrix}\left(\frac{2}{N}\right)\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

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$$= \left(\frac{4}{N^2}\right)\begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

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$$= \left(\frac{4}{N^2}\right)\begin{bmatrix} \sum\limits_T \hat{u}^2 & \sum\limits_T \hat{u}^2 - \sum\limits_C \hat{u}^2 \\ \\ 0 & \\ \\ \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

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CT p.75: DOF correction w/ empirical (not theoretical) basis, N/(N-K)

All formulas together, before plugging in K.

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Estimated coefficients:

$$\hat{\beta} = \begin{bmatrix} \bar{Y}_T - \bar{Y}_C \\ \\ \bar{Y}_C \end{bmatrix}$$

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Estimated VCV matrix under homoskedasticity:

$$\left(\frac{2}{N(N-K)}\right) \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \left(\sum_T \hat{u}^2 + \sum_C \hat{u}^2\right)$$

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$$\left(\frac{4}{N(N-K)}\right) \begin{bmatrix} \sum_T \hat{u}^2 + \sum_C \hat{u}^2 & -\sum_C \hat{u}^2 \\ \\ -\sum_C \hat{u}^2 & \sum_C \hat{u}^2 \end{bmatrix}$$

All formulas together.

Estimated coefficients:

$$\hat{\beta} = \begin{bmatrix} \bar{Y}_T - \bar{Y}_C \\ \\ \bar{Y}_C \end{bmatrix}$$

Estimated VCV matrix under homoskedasticity:

$$\left(\frac{2}{N(N-2)}\right) \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \left(\sum_T \hat{u}^2 + \sum_C \hat{u}^2\right)$$

Estimated VCV matrix under heteroskedasticity:

$$\left(\frac{4}{N(N-2)}\right) \begin{bmatrix} \sum_T \hat{u}^2 + \sum_C \hat{u}^2 & -\sum_C \hat{u}^2 \\ \\ -\sum_C \hat{u}^2 & \sum_C \hat{u}^2 \end{bmatrix}$$

General case, treating fraction p.

$$(X'X) = N \begin{bmatrix} p & p \\ p & 1 \end{bmatrix}$$

$$(X'X)^{-1} = \frac{1}{p(1-p)N} \begin{bmatrix} 1 & -p \\ -p & p \end{bmatrix}$$

All formulas together, general case.

Estimated coefficients:

$$\hat{\beta} = \begin{bmatrix} \bar{Y}_T - \bar{Y}_C \\ \\ \bar{Y}_C \end{bmatrix}$$

Estimated VCV matrix under homoskedasticity:

$$\left(\frac{1}{p(1-p)N(N-2)}\right) \begin{bmatrix} 1 & -p \\ -p & p \end{bmatrix} \left(\sum_T \hat{u}^2 + \sum_C \hat{u}^2\right)$$

Estimated VCV matrix under heteroskedasticity:

$$\left(\frac{1}{p^2(1-p)^2N(N-2)}\right) \begin{bmatrix} (1-p)^2 \sum_T \hat{u}^2 + p^2 \sum_C \hat{u}^2 & -p^2 \sum_C \hat{u}^2 \\ \\ -p^2 \sum_C \hat{u}^2 & p^2 \sum_C \hat{u}^2 \end{bmatrix}$$

Focus on the coefficient on treatment.

Estimated coefficient: $\hat{\beta}_1 = \bar{Y}_T - \bar{Y}_C$

Estimated variance of $\hat{\beta}_1$ under homoskedasticity:

$$\frac{\sum_T \hat{u}^2 + \sum_C \hat{u}^2}{p(1-p)N(N-2)}$$

Estimated variance of $\hat{\beta}_1$ under heteroskedasticity:

$$\frac{(1-p)^2 \sum_T \hat{u}^2 + p^2 \sum_C \hat{u}^2}{p^2(1-p)^2 N(N-2)}$$

Focus on the coefficient on treatment.

Estimated coefficient: $\hat{\beta}_1 = \bar{Y}_T - \bar{Y}_C$

Estimated variance of $\hat{\beta}_1$ under homoskedasticity:

$$\frac{\sum_T \hat{u}^2 + \sum_C \hat{u}^2}{p(1-p)N(N-2)} = \frac{\sum_T \hat{u}^2 + \sum_C \hat{u}^2}{(N-2)} \left(\frac{1}{p(1-p)N}\right)$$

Estimated variance of $\hat{\beta}_1$ under heteroskedasticity:

$$\frac{(1-p)^2 \sum_T \hat{u}^2 + p^2 \sum_C \hat{u}^2}{p^2(1-p)^2 N(N-2)}$$

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Estimated variance of $\hat{\beta}_1$ under homoskedasticity:

$$\frac{\sum_T \hat{u}^2 + \sum_C \hat{u}^2}{p(1-p)N(N-2)} = \frac{\sum_T \hat{u}^2 + \sum_C \hat{u}^2}{(N-2)} \left(\frac{1}{pN} + \frac{1}{(1-p)N} \right)$$

Estimated variance of $\hat{\beta}_1$ under heteroskedasticity:

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$$\frac{(1-p)^2 \sum_T \hat{u}^2 + p^2 \sum_C \hat{u}^2}{p^2(1-p)^2 N(N-2)} =$$

$$\frac{\sum_T \hat{u}^2}{p^2 N(N-2)} + \frac{\sum_C \hat{u}^2}{(1-p)^2 N(N-2)}$$

Focus on the coefficient on treatment.

Estimated coefficient: $\hat{\beta}_1 = \bar{Y}_T - \bar{Y}_C$

Estimated variance of $\hat{\beta}_1$ under homoskedasticity:

$$\frac{\sum_T \hat{u}^2 + \sum_C \hat{u}^2}{p(1-p)N(N-2)} = \frac{\sum_T \hat{u}^2 + \sum_C \hat{u}^2}{(N-2)} \left(\frac{1}{pN} + \frac{1}{(1-p)N} \right)$$

Estimated variance of $\hat{\beta}_1$ under heteroskedasticity:

$$\frac{(1-p)^2 \sum_T \hat{u}^2 + p^2 \sum_C \hat{u}^2}{p^2(1-p)^2 N(N-2)} =$$

$$\frac{\sum_T \hat{u}^2}{p(N-2)} \left(\frac{1}{pN} \right) + \frac{\sum_C \hat{u}^2}{(1-p)(N-2)} \left(\frac{1}{(1-p)N} \right)$$

Focus on the coefficient on treatment.

Estimated coefficient: $\hat{\beta}_1 = \bar{Y}_T - \bar{Y}_C$

Estimated variance of $\hat{\beta}_1$ under homoskedasticity:

$$\frac{\sum_T \hat{u}^2 + \sum_C \hat{u}^2}{p(1-p)N(N-2)} = \underbrace{\frac{\sum_T \hat{u}^2 + \sum_C \hat{u}^2}{(N-2)}}_{\text{variance}} \left(\frac{1}{pN} + \frac{1}{(1-p)N} \right)$$

Estimated variance of $\hat{\beta}_1$ under heteroskedasticity:

$$\frac{(1-p)^2 \sum_T \hat{u}^2 + p^2 \sum_C \hat{u}^2}{p^2(1-p)^2 N(N-2)} =$$

$$\underbrace{\frac{\sum_T \hat{u}^2}{p(N-2)}}_{\text{variance}} \left(\frac{1}{pN} \right) + \underbrace{\frac{\sum_C \hat{u}^2}{(1-p)(N-2)}}_{\text{variance}} \left(\frac{1}{(1-p)N} \right)$$

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$$\frac{\sum_T \hat{u}^2 + \sum_C \hat{u}^2}{p(1-p)N(N-2)} = \underbrace{\frac{\sum_T \hat{u}^2 + \sum_C \hat{u}^2}{(N-2)}}_{variance} \left(\frac{1}{pN} \underbrace{+}_{difference\ of} \frac{1}{(1-p)N} \right)$$

Estimated variance of $\hat{\beta}_1$ under heteroskedasticity:

$$\frac{(1-p)^2 \sum_T \hat{u}^2 + p^2 \sum_C \hat{u}^2}{p^2(1-p)^2 N(N-2)} =$$

$$\underbrace{\frac{\sum_T \hat{u}^2}{p(N-2)}}_{variance} \left(\frac{1}{pN} \right) \underbrace{+}_{difference\ of} \underbrace{\frac{\sum_C \hat{u}^2}{(1-p)(N-2)}}_{variance} \left(\frac{1}{(1-p)N} \right)$$

Focus on the coefficient on treatment.

Estimated coefficient: $\hat{\beta}_1 = \bar{Y}_T - \bar{Y}_C$

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$$\frac{\sum_T \hat{u}^2 + \sum_C \hat{u}^2}{p(1-p)N(N-2)} = \underbrace{\frac{\sum_T \hat{u}^2 + \sum_C \hat{u}^2}{(N-2)}}_{variance} \left(\underbrace{\frac{1}{pN}}_{averages} \underbrace{+}_{difference\ \ of} \underbrace{\frac{1}{(1-p)N}}_{averages} \right)$$

Estimated variance of $\hat{\beta}_1$ under heteroskedasticity:

$$\frac{(1-p)^2 \sum_T \hat{u}^2 + p^2 \sum_C \hat{u}^2}{p^2(1-p)^2 N(N-2)} =$$

$$\underbrace{\frac{\sum_T \hat{u}^2}{p(N-2)}}_{variance} \underbrace{\left(\frac{1}{pN}\right)}_{averages} \underbrace{+}_{difference\ \ of} \underbrace{\frac{\sum_C \hat{u}^2}{(1-p)(N-2)}}_{variance} \underbrace{\left(\frac{1}{(1-p)N}\right)}_{averages}$$