ECON 626: Empirical Microeconomics

DD with Variation in Treatment Timing

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The do file `ECON626-L3-A3-timing.do` generates a simulated panel data set with $N = 100$ units observed over $T = 100$ periods, $t = 1, \ldots, 100$. The data set contains three timing groups. Group A accounts for one quarter of the sample; units in Group A begin receiving treatment at $t = 41$. Group B accounts for one quarter of the sample; units in Group B begin receiving treatment at $t = 81$. Units in Group C are never treated.

1. Before turning to the Stata program, it is helpful to calculate the weighting factors by hand.

   (a) Use the information above to complete the table:

Group	**Parameter**	**Value**
A	n_A	
B	n_B	
C	n_C	
A	$\bar{D}_A$	
B	$\bar{D}_B$	

   (b) With three timing groups including one that is never treated, there are four 2×2 DD components: AC, BC, AB, and BA. Each treated vs. never-treated comparison receives weight:

$$s_{jC} = \left[(n_j + n_C)^2 n_{jC} \left(1 - n_{jC}\right) \bar{D}_j (1 - \bar{D}_j)\right] / \hat{V}^{\tilde{D}} \text{ for } j = A, B \tag{1}$$

where $\hat{V}^D$ is the variance of the fixed-effects-adjusted treatment dummy (more on this later) and $n_{jC} = n_j/(n_j + n_C)$. The formulas for the weights on the timing group comparisons are slightly more complicated:

$$s_{AB} = \left[\left((n_A + n_B)\left(1 - \bar{D}_B\right)\right)^2 n_{AB}(1 - n_{AB})\left(\frac{\bar{D}_A - \bar{D}_B}{1 - \bar{D}_B}\right)\left(\frac{1 - \bar{D}_A}{1 - \bar{D}_B}\right)\right]/\hat{V}^{\tilde{D}} \tag{2}$$

$$s_{BA} = \left[\left((n_A + n_B)\,\bar{D}_A\right)^2 n_{AB}(1 - n_{AB})\frac{\bar{D}_B}{\bar{D}_A}\left(\frac{\bar{D}_A - \bar{D}_B}{\bar{D}_A}\right)\right]/\hat{V}^{\tilde{D}} \tag{3}$$

We don't have a simple formula for $\hat{V}^{\tilde{D}}$, but we know that the weights s_{AC}, s_{BC}, s_{AB}, and s_{BA} sum to one. This allows us to ignore $\hat{V}^{\tilde{D}}$ and calculate the numerator for each of the weighting factors. The tables below may be helpful in doing this.

$(n_A + n_C)^2$	$* \; n_{AC}$	$* \; (1 - n_{AC})$	$* \; \bar{D}_A$	$* \; (1 - \bar{D}_A)$	$= s_{AC} * \hat{V}^{\tilde{D}}$
______	______	______	______	______	= ______

$(n_B + n_C)^2$	$* \; n_{BC}$	$* \; (1 - n_{BC})$	$* \; \bar{D}_B$	$* \; (1 - \bar{D}_B)$	$= s_{BC} * \hat{V}^{\tilde{D}}$
______	______	______	______	______	= ______

$\left[(n_A + n_B)\left(1 - \bar{D}_B\right)\right]^2$	$* \; n_{AB}(1 - n_{AB})$	$* \; \left(\frac{\bar{D}_A - \bar{D}_B}{1 - \bar{D}_B}\right)$	$* \; \left(\frac{1 - \bar{D}_A}{1 - \bar{D}_B}\right)$	$= s_{AB} * \hat{V}^{\tilde{D}}$
______	______	______	______	= ______

$\left[(n_A + n_B)\,\bar{D}_A\right]^2$	$* \; n_{AB}(1 - n_{AB})$	$* \; \left(\frac{\bar{D}_B}{\bar{D}_A}\right)$	$* \; \left(\frac{\bar{D}_A - \bar{D}_B}{\bar{D}_A}\right)$	$= s_{AB} * \hat{V}^{\tilde{D}}$
______	______	______	______	= ______

(c) Now calculate the actual weights s_{AC}, s_{BC}, s_{AB}, and s_{BA} by normalizing the weights calculated above so that they sum to one.

(d) Consider a case here the treatment effect **on Group A** is equal to 10 and the treatment effect **on Group B** was equal to 5. In this context, what are the expected values of the estimated $\hat{\beta}$ in the four 2×2 DD estimates if the common trends assumption holds? (Hint: do not over-think this question; it is intended to be straightforward.)

$$\hat{\beta}^{DD}_{AC} = \underline{\qquad\qquad}$$

$$\hat{\beta}^{DD}_{BC} = \underline{\qquad\qquad}$$

$$\hat{\beta}^{DD}_{AB} = \underline{\qquad\qquad}$$

$$\hat{\beta}^{DD}_{BA} = \underline{\qquad\qquad}$$

(e) What is the expected value of $\hat{\beta}^{DD}$ from the fixed-effects DD model?

2. Now run `ECON626-L3-A3-timing.do`. Does the estimated coefficient match what you calculated in (1e)? Why or why not?

3. In real life, we would like to use a more efficient approach to calculating s_{AC}, s_{BC}, s_{AB}, and s_{BA}. This approach relies on the fact that the numerators in the weights are a product of two components: (i) a term reflecting the sample size of each 2×2 DD (as a share of NT) and (ii) the fixed-effects-adjusted variance of the treatment variable within each 2×2 DD sample. So, for example,

$$s_{jC} = [(n_j + n_C)^2 \underbrace{n_{jC}\,(1 - n_{jC})\,\bar{D}_j(1 - \bar{D}_j)}_{\hat{V}^{\tilde{D}}_{jC}}]/\hat{V}^{\tilde{D}} \tag{4}$$

while

$$s_{AB} = \left((n_A + n_B)\left(1 - \bar{D}_B\right)\right)^2 \underbrace{n_{AB}(1 - n_{AB})\left(\frac{\bar{D}_A - \bar{D}_B}{1 - \bar{D}_B}\right)\left(\frac{1 - \bar{D}_A}{1 - \bar{D}_B}\right)}_{\hat{V}^{\tilde{D}}_{AB}} / \hat{V}^{\tilde{D}} \tag{5}$$

and

$$s_{BA} = \left((n_A + n_B)\,\bar{D}_A\right)^2 \underbrace{n_{AB}(1 - n_{AB})\frac{\bar{D}_B}{\bar{D}_A}\left(\frac{\bar{D}_A - \bar{D}_B}{\bar{D}_A}\right)}_{\hat{V}^{\tilde{D}}_{BA}} / \hat{V}^{\tilde{D}} \tag{6}$$

The last part of `ECON626-L3-A3-timing.do` calculates $\hat{V}^{\tilde{D}}$ for the entire sample (i.e. calculates the denominator in the DD weights) and the constructs dummy variables for each of the four 2×2 DD timing groups. Extend this code to calculate the fixed-effects-adjusted variance of treatment in each timing group, and use these variances to derive the DD weights in Stata. Confirm that these weights match those derived above.